Startups and Upstarts: Disadvantageous Information in R&D*

Yu Awaya† and Vijay Krishna‡

October 16, 2019

Abstract

We study a continuous-time R&D race between an established firm and a startup under asymmetric information. R&D investment brings success stochastically but only if the innovation is feasible. The only asymmetry between the firms is that the established firm has better information about the feasibility of the innovation. We show that there is an equilibrium in which the poorly-informed startup wins *more* often, and has higher expected profits, than the better-informed incumbent. When the informational asymmetry is large, this is the *unique* equilibrium outcome. The channel by which better information becomes a competitive disadvantage appears to be new—it does not stem from a negative value of information or from a second-mover advantage. Rather, it stems from the fact that better information dulls the incentive to learn from one's rival.

*The research reported here was supported by a grant from the National Science Foundation (SES-1626783). We are grateful to Nageeb Ali, Gaurab Aryal, Kaustav Das, Erik Madsen, John Morgan, Tsz-Ning Wong and Shmuel Zamir for helpful discussions.

†Department of Economics, University of Rochester, E-mail: yuawaya@gmail.com.

‡Department of Economics, Penn State University, E-mail: vkrishna@psu.edu.

1 Introduction

Why Tesla and not GM or Toyota? Why Amazon and not Sears or Wal-Mart? Why are startups the source of so many innovations instead of, and at the expense of, established firms? In his history of the hard-disk industry over two decades, Christiansen (1997) found that the market for each new generation of disk drives—typically, smaller in size—was dominated by a different set of firms. Of the 17 firms in the industry in 1976, only IBM's disk-drive division survived until 1995. In the same period, there were 129 entrants but 109 of these failed to make the transition to later generations (Christiansen, 1997, p. 22). Many technological innovations came from startups.

What advantage does a startup have over an established firm? In one of his many classics, Arrow (1962) argued that because of the "monopolist's disincentive created by his preinvention profits" (p. 622) an entrant would have more to gain from an innovation. This is sometimes called the "replacement effect" because by successfully innovating, the monopolist would only be replacing himself while the entrant would be replacing the monopolist. Running counter to Arrow's reasoning are the strong incentives that an incumbent has to protect its monopoly position. This stems from the Econ 101 $m > 2d$ inequality—monopoly profits exceed total profits in a duopoly—which can be cleverly rearranged as $m - d > d$. In this form, it says that the incentive of the incumbent to preserve its monopoly is greater than the incentive of the startup to enter as a duopolist (Gilbert and Newberry, 1982). This "preemption effect" is at odds with the replacement effect. There are other forces that may favor incumbents as well—it may be better at R&D or have deeper pockets. Whether the balance of all these forces favors incumbents or startups is then an empirical question. In a recent paper, Igami (2017) went back to the disk-drive industry and constructed a structural model to try to answer this question. A large fraction of firms failed to make the transition from 5.25- to 3.5-inch drives. Igami found evidence that Arrow's replacement effect played a substantial role, explaining about 60% of the turnover.

In this paper, we study a continuous-time, winner-take-all R&D race between an established firm and a startup. Firms must invest to engage in R&D on a project whose feasibility is unclear. Precisely, there are two states of nature. In one, the innovation is feasible and R&D brings success stochastically—in the manner of exponential bandits. In the other state, the innovation is infeasible. Firms do not know the state but receive informative private signals about it. The only difference between the firms is that the established firm's signal is more accurate than that of the startup. As the race proceeds, lack of success causes both firms to become increasingly pessimistic about the feasibility of the innovation. Each must then decide when to quit, a decision that is observed by its rival[1] and is irrevocable[2]. Such models are rather standard—the basic structure originates in Choi (1991) and has been studied by others—and the only new ingredient we add is comparable asymmetric information.

We identify an entirely new effect that, like the replacement effect, works to the detriment of the established firm—better information is a competitive disadvantage and as a result, the less-informed entrant succeeds more often and has a higher payoff than the better-informed incumbent. To isolate this new effect, we assume that the firms are alike in all other respects—the costs and benefits of R&D as well as their R&D abilities are the same. Because the gains from R&D are the same, the replacement and preemption effects are absent. This is done deliberately in order to isolate the effects of asymmetric information.[3]

Our main result is

Theorem 1 *There is an equilibrium of the R&D game in which the less-informed startup wins* more *often, and has a* higher *payoff, than the better-informed incumbent. Moreover, if the quality of the incumbent's information is* much *better than that of the startup, then this is the* only *equilibrium.*

Our result shows that in an otherwise symmetric situation, the incumbent's informational advantage becomes a competitive disadvantage—it wins the R&D race less often than the startup and, as we will see, has a lower payoff as well. The startup is favored to win precisely because it is less informed!

We call this an "upstart equilibrium." In such an equilibrium, the less-informed startup is, quite naturally, willing to learn from the incumbent. But because of its superior information, the incumbent is unwilling to learn from the startup/upstart. The learning is so unbalanced that the startup gains an advantage over the incumbent. The incumbent suffers from a "curse of knowledge"—its superior knowledge hinders learning.

Precisely, both the incumbent and the startup play strategies that reveal over time whether or not they are optimistic. But since the incumbent's information is of higher quality than that of the startup, when pessimistic it exits early in the race based solely on its own information. The reason is that while the startup also reveals its signal during the play of the game, this comes too late to make it worthwhile for a pessimistic incumbent to stay and learn. On the other hand, the information does not

[1] Pharmaceutical companies must register drug trials with the Food and Drug Administration and report progress or lack thereof publicly. In other industries, R&D activity must be reported to investors.

[2] We base this on the assumption that once a project is abandoned, it is prohibitively costly to restart it. This seems closer to reality than assuming that firms can shut down R&D and then costlessly restart it.

[3] As we show below, the equilibrium studied here is robust and introducing small asymmetries in payoffs, R&D costs or abilities would not overturn the results.

come too late for the optimistic incumbent for whom it is worthwhile to stay and learn the startup's signal. Thus a pessimistic incumbent exits early while an optimistic one stays. This means that the startup can learn the incumbent's information at low cost. During the play of the game, both the optimistic and the pessimistic startup learn the incumbent's information but only the optimistic incumbent learns the startup's information.

It is then not too hard to argue that if both firms are optimistic or both are pessimistic, they exit at the same time.[4] The same is true when the incumbent is optimistic and the startup pessimistic—this is because they both learn each other's signal. The remaining case is one with a pessimistic incumbent and an optimistic startup. The incumbent exits early and so the startup learns that it is pessimistic. But its own optimism causes the startup to continue with R&D nevertheless. Now the startup has a *greater* chance of winning than does the incumbent.

The upstart equilibrium outcome has some salient features. While it can be supported as a perfect Bayesian equilibrium, it does not rely on any particular choice of off-equilibrium beliefs. More important, when the informational advantage of the incumbent is large, it is the unique Nash equilibrium outcome. The formal argument relies on the iterated elimination of dominated strategies—in our game, this procedure leaves a single outcome. Some idea of the reasoning can be gauged by noting that in these circumstances there cannot be a "mirror equilibrium" in which the roles of the two firms are reversed and the incumbent learns more from the startup than the other way around. Because the startup's information is of very poor quality, it is not worthwhile for the incumbent to invest in learning this. So when the startup's information is very poor, a mirror equilibrium does not exist. In our formal analysis, we rule out not only the mirror equilibrium but all others as well.

A new information "paradox"? Intuition suggests that information should confer a strategic advantage. In our model, it is a disadvantage. The fact that information can have paradoxical consequences in multi-person settings is, of course, well-understood and one may rightly wonder whether our main result is just a manifestation of a known phenomenon.

It is known that information may have a negative value. Hirshleifer (1971) showed that publicly available information may make all agents worse off ex ante. There are also examples of games in which information privately available to an agent reduces his or her ex ante payoff (see Bassan et al., 2003 or Maschler et al., 2013 for examples). One might rightly wonder then whether, in the game we study, the value of private information is negative as well. This is not the case. We show below that in the upstart equilibrium, the value of information is *positive* for both firms—each firm's expected payoff is increasing in the quality of its own information. Theorem 1 above

[4] Around 2006, a new kind of technology for television display screens—emission display—seemed very promising. The main competitors to develop this were Sony and a Canon/Toshiba joint venture. Sony abandoned the technology publicly in 2009 and other firms followed soon after.

is a comparison of payoffs across firms and does not contradict the fact that each firm has the individual incentive to become better informed.

It is also known that in many situations, there is a second-mover advantage. Does the competitive advantage of the startup stem from the fact that it moves second and is able to learn the incumbent's information? This is not the case either. In our model, the order of moves (when to exit) is not specified exogenously; rather it is determined in equilibrium. For some signal realizations, the incumbent exits early and the startup learns from the incumbent. In others, the startup exits early and the incumbent learns from the startup.

In our model, the incumbent has a competitive disadvantage because its superior information dulls its incentives to learn relative to the startup. The marginal value to the incumbent of learning the startup's poor information is smaller than the marginal value to the startup of learning the incumbents better information. The startup is willing to wait-and-see whereas the incumbent is not. But how do we know that this is the reason for the "paradox"? In Section 8 we study a variant of the main model in which firms cannot observe each other's exit decisions. All other aspects of the model remain unchanged but now there is no possibility of learning from each other. We show that once there is no possibility of learning, the "paradox" disappears—in the *unique* equilibrium, the expected payoff of the better-informed firm is now higher than that of its rival.

Overconfidence The popular press is full of stories of brash Silicon Valley entrepreneurs who embark on risky projects that established firms deem unworthy. Most of these startups fail but some do succeed and perhaps lead to the kinds of disruption that is observed. Some studies have argued that this over-investment in risky projects stems not from risk-loving preferences but rather from overconfidence.[5] As one observer of the startup phenomenon has written:

> "In the delusions of entrepreneurs are the seeds of technological progress."
> (Surowiecki, 2014)

In this view, the Elon Musks of the world drive innovation because of unwarranted self-confidence. They remain optimistic in environments that the GMs of the world are pessimistic about, and perhaps realistically so.

Even though our model and analysis has no behavioral or psychological elements, it can be seen as providing a rational reinterpretation of such behavior. When the incumbent firm's information is not favorable to the project while the startup's is, the former is pessimistic and the latter optimistic. The startup invests in R&D while the better-informed incumbent does not. In these circumstances, the rational optimism of the startup would be observationally equivalent to overconfidence. In single-person

[5] See, for example, Wu and Knott (2006). Another study found that entrepreneurs are prone to overestimate their own life spans relative to the rest of the population (Reitveld et al. 2013)!

problems, Benoît and Dubra (2011) argued that in many situations a fully rational Bayesian agent may end up with beliefs that, to an outside observer, would seem overconfident. They showed that this "apparent overconfidence" could be generated solely by the structure of information available to the agent. Our model and equilibrium can be interpreted as doing the same, but now in a strategic situation with more than one agent. The postulated information structure and the upstart equilibrium results in behavior that an outside observer may well attribute to overconfidence.

The incumbent may also be a victim of apparent overconfidence—it is so sure of its own information that it does not find it worthwhile to try to learn what the startup knows. This is the main driving force of our result but again, its basis is not psychological. Rather, it is the result of rational calculation.

Related literature The basic model of this paper is rather standard. R&D races where the arrival times of success are exponentially distributed and there is uncertainty about the arrival rates were first studied by Choi (1991). Malueg and Tsutsui (1997) extend Choi's model to allow for flexibility in the intensity of R&D. In a variant of Choi's model, Wong (2018) examines the consequences of imperfect patent protection thereby relaxing the winner-take-all structure common to most of the literature.[6] Chatterjee and Evans (2004) introduce another kind of uncertainty—there are two alternative paths to success and it is not known which is the correct one. Firms may switch from one path to another based on their beliefs. Das and Klein (2018) study a similar model and show that there is a unique Markov perfect equilibrium which is efficient when firms are symmetric in R&D ability and not otherwise.

In all of these models, however, there is no asymmetry of information—firms' equilibrium beliefs are identical. In our model, firms receive private signals prior to the race and the resulting asymmetry of beliefs is the key to our results.

The model of Moscarini and Squintani (2010) is, in its basic structure, most closely related to ours. These authors study a very general set-up with arbitrary distributions of arrival times (not necessarily exponential), continuous signals and differing costs and benefits of R&D. They show the possibility that the exit of one firm leads the other to regret staying as long—the firm suffers from a "survivor's curse"—and so it also exits as soon as possible. Our model differs from that of Moscarini and Squintani in that we have discrete states and signals. At the same time, it specializes their model by assuming exponentially distributed arrival times, identical costs and benefits of R&D and *comparable* information. Moscarini and Squintani also point to a "quitter's curse"—regret at exiting too early. When the firms' information is comparable, as we assume, even the curses are asymmetrically distributed. In equilibrium, the better-informed firm is subject to both curses while the less-informed firm is never subject to the survivor's curse. Finally, we derive

[6] In Wong's model the feasibility of the projects is independent across firms and so one firm cannot learn from the other firm's lack of success. In our model, and the others mentioned, the feasibility is perfectly correlated.

circumstances in which there is a unique equilibrium outcome and these too depend on the relative quality of the firms' information.

Strategic experimentation Our model is related to those of strategic experimentation, especially with exponential bandits as in Keller, Rady and Cripps (2005). Unlike our model, the latter are not winner-take-all as one person's success does not preclude the other's. Also, in these models it is possible to switch back and forth between the risky and safe arms, unlike the irrevocable exit assumption we make. Strategic experimentation models typically have multiple Markov equilibria whereas ours has a unique Nash equilibrium. Another difference is that whereas equilibria of strategic experimentation models display under-investment relative to a planner's solution—there is free-riding—in our model firms over-invest.

While most of these models were studied under symmetric information, in recent work, Dong (2018) has studied a variant with asymmetric and comparable information—one person has a private signal but the other is completely uninformed.[7] In the equilibrium she studies, this asymmetry induces more experimentation than if the situation were symmetric.

Wars of attrition Our model also shares important features with the war of attrition—in particular, the winner-take-all and irrevocable exit assumptions. There is, of course, a vast literature on wars of attrition with and without incomplete information. A related paper in this vein is by Chen and Ishida (2017), who study a model which combines elements from strategic experimentation with wars of attrition. As in strategic experimentation models, one firm's successful innovation does not preclude successful innovation by the other firm. As in the war of attrition, exit by one firm ends the game. Firms are asymmetric in how efficient they are at R&D. There is a mixed strategy equilibrium and Chen and Ishida (2017) exhibit the possibility that the less efficient firm may win more often.

The remainder of the paper is organized as follows. The model of an R&D race is outlined in the next section. Section 3 studies, as a benchmark, the case of a single firm without competition. There is no surprise here—if alone, the better informed firm is more likely to succeed than the less informed firm. In Section 4 we then study the case of two competing firms and exhibit the upstart equilibrium mentioned above. Section 5 shows that this equilibrium is unique when the asymmetry in the quality of information is large. In Section 6, we show that despite the fact that in equilibrium the less informed firm wins more often, the value of information is positive for both firms. Section 7 shows that the main result generalizes when the firms may get more than two signals and so have finer information. In Section 8 we study a variant of the model in which firms' exits decisions are unobserved. We show that in the

[7] Klein and Wagner (2018) study a bandit problem where the quality of information of the players is the same.

unique equilibrium of this variant, the better-informed firm has a higher payoff than the less-informed firm. This shows that the reason that information is a competitive disadvantage in the main model is that firms can learn from each other and that the better-informed firm has a smaller incentive to learn. A comparison to the planner's optimal solution is made in Section 9. Section 10 concludes. Appendix A considers the special case when there is no asymmetric information and the firms hold common beliefs throughout while Appendix B contains some auxiliary results about the model with unobserved exit.

2 Preliminaries

Two firms compete in an R&D race to produce an innovation. Time runs continuously, the horizon is infinite and the interest rate is $r > 0$. The firm that succeeds first will obtain a patent that yields flow monopoly profits of m forever after. Each firm decides on how long it wants to actively participate in the race, if at all, and must incur a flow cost of c while it is active. A firm only chooses whether or not to be active, and not its intensity of R&D. Once a firm quits, it cannot rejoin the race. Also, if a firm quits at time t, say, then this is immediately observed by the other firm.[8] The game ends either if one of the firms succeeds or once both firms quit.

Whether or not the innovation is worth pursuing is uncertain, however, and depends on an unknown state of nature that may be G ("good") or B ("bad") with prior probabilities π and $1-\pi$, respectively. In state B, the innovation is not technologically feasible and all R&D activity is futile. In state G, it is feasible and success arrives at a Poisson rate $\lambda > 0$ per instant, independently for each firm provided, of course, that the firm is still active. This means that the distribution of arrival times of success is *exponential*, that is, the probability that in state G a firm will succeed before time t is $1 - e^{-\lambda t}$.

The two firms are alike in all respects but one—firm 1 (the "incumbent" or established firm) is better informed about the state of nature, G or B, than is firm 2 (the "startup" or entrant firm). Specifically, before the race starts, each firm i receives a noisy private signal $s_i \in \{g_i, b_i\}$ about the state. Conditional on the state, the signals of the two firms are independent and

$$\Pr[g_i \mid G] = \Pr[b_i \mid B] = q_i > \tfrac{1}{2}$$

We will refer to q_i as the *quality* of i's signal or information.[9] Throughout, we will assume that firm 1's signal is of higher quality than that of firm 2 in the sense that $q_1 > q_2$ and so firm 1 is better informed.

[8] This could happen with a delay $\Delta > 0$ so that if a firm quits at time t, the other firm learns of this only at time $t + \Delta$. We have chosen to set $\Delta = 0$ to simplify the exposition but our analysis is robust to the case when Δ is small (details are available from the authors).

[9] The assumption that $\Pr[g_i \mid G] = \Pr[b_i \mid B]$ is made only for simplicity. It would be enough to assume that firm 1's signals were more informative than firm 2's signals in the sense of Blackwell.

Denote by $p(s_i)$ the posterior probability that the state is G conditional on the signal s_i, that is,

$$p(s_i) = \Pr[G \mid s_i]$$

and similarly, denote by $p(s_1, s_2)$ the posterior probability that the state is G conditional on the signals (s_1, s_2), that is,

$$p(s_1, s_2) = \Pr[G \mid s_1, s_2]$$

It is easy to see that since firm 1's signal is more accurate than firm 2's signal, that is, $q_1 > q_2$,

$$p(b_1, b_2) < p(b_1, g_2) < p(g_1, b_2) < p(g_1, g_2) \tag{1}$$

It is useful to define p^* to be such that if a firm believes that the probability that the state is G is p^*, then the flow expected gain is the same as the flow cost. Thus, p^* is defined by

$$\underbrace{p^*\lambda}_{\text{success rate}} \times \underbrace{\frac{m}{r}}_{\text{gain}} = \underbrace{c}_{\text{cost}}$$

and so

$$p^* = \frac{rc}{\lambda m} \tag{2}$$

and we will suppose that $0 < p^* < 1$.

The following definition will prove useful in the subsequent analysis. Suppose both firms have a common belief at time 0 that the probability of state G is p_0 and with this belief both engage in R&D at time 0. As time elapses and both firms are active but neither firm has been successful, the firms become increasingly pessimistic that the state is G and the posterior probability that the state is G decreases. At time t, the common belief p_t is such that[10]

$$\frac{p_t}{1-p_t} = e^{-2\lambda t}\frac{p_0}{1-p_0}$$

since, conditional on the state being G, the probability that neither firm has been successful until time t is $e^{-2\lambda t}$.

Definition 1 *If the initial belief $p_0 > p^*$, $T(p_0)$ is the time when, absent any success by either firm, this belief will decay to p^*, that is,*

$$e^{-2\lambda T(p_0)}\frac{p_0}{1-p_0} = \frac{p^*}{1-p^*} \tag{3}$$

If the initial belief $p_0 \leq p^*$, then $T(p_0) = 0$.

[10] This is just Bayes' rule in terms of odds ratios: given any event $\mathcal{E}$, we have

$$\frac{\Pr[G \mid \mathcal{E}]}{\Pr[B \mid \mathcal{E}]} = \frac{\Pr[\mathcal{E} \mid G]}{\Pr[\mathcal{E} \mid B]} \times \frac{\Pr[G]}{\Pr[B]}$$

Equivalently, for $p_0 > p^*$,

$$T(p_0) = \frac{1}{2\lambda} \ln\left(\frac{p_0}{1-p_0}\right) - \frac{1}{2\lambda} \ln\left(\frac{p^*}{1-p^*}\right)$$

To save on notation, we will write

$$T(s_i) \equiv T(p(s_i)) \tag{4}$$

and

$$T(s_1, s_2) \equiv T(p(s_1, s_2)) \tag{5}$$

3 Single-firm benchmark

Before studying the situation in which the two firms are competing against one another, it is useful to consider the case where each firm acts in isolation. Comparing the situation in which firm 1 is alone to the situation in which firm 2 is alone, we obtain

Proposition 0 *The probability that firm* 1 *is successful when alone is greater than the probability that firm* 2 *is successful when alone.*

To establish the proposition, first note that if firm i gets a signal $s_i \in \{g_i, b_i\}$, then its belief that the state is G is $p(s_i)$ at time 0. If $p(s_i) \leq p^*$ then the firm should not engage in R&D at all since its expected profits from R&D are non-positive. But if $p(s_i) > p^*$ then it is worthwhile to engage in R&D at time 0 and continue to do so as long as its belief $p_t(s_i)$ at time t remains above p^*. In terms of odds ratios, this means that a solitary firm should remain active as long as

$$\frac{p_t(s_i)}{1-p_t(s_i)} = e^{-\lambda t} \frac{p(s_i)}{1-p(s_i)} > \frac{p^*}{1-p^*}$$

reflecting the fact that the probability that a single firm does not succeed until time t is just $e^{-\lambda t}$. The following result is immediate.

Lemma 3.1 *A single firm with signal s_i should quit at the earliest time t such that $p_t(s_i) \leq p^*$.*

Proof. If firm i with signal s_i quits at time t_i, its flow profit is

$$r \int_0^{t_i} e^{-rt} \Pr[\mathcal{S}_0(t)] \left(p_t(s_i) \frac{\lambda m}{r} - c\right) dt = \lambda m \int_0^{t_i} e^{-rt} \Pr[\mathcal{S}_0(t)] (p_t(s_i) - p^*) \, dt \tag{6}$$

where $\Pr[\mathcal{S}_0(t)] = e^{-\lambda t} p(s_i) + 1 - p(s_i)$ is the probability that there has been no success until time t. Recall that $p^* = rc/\lambda m$. The result obviously follows. ■

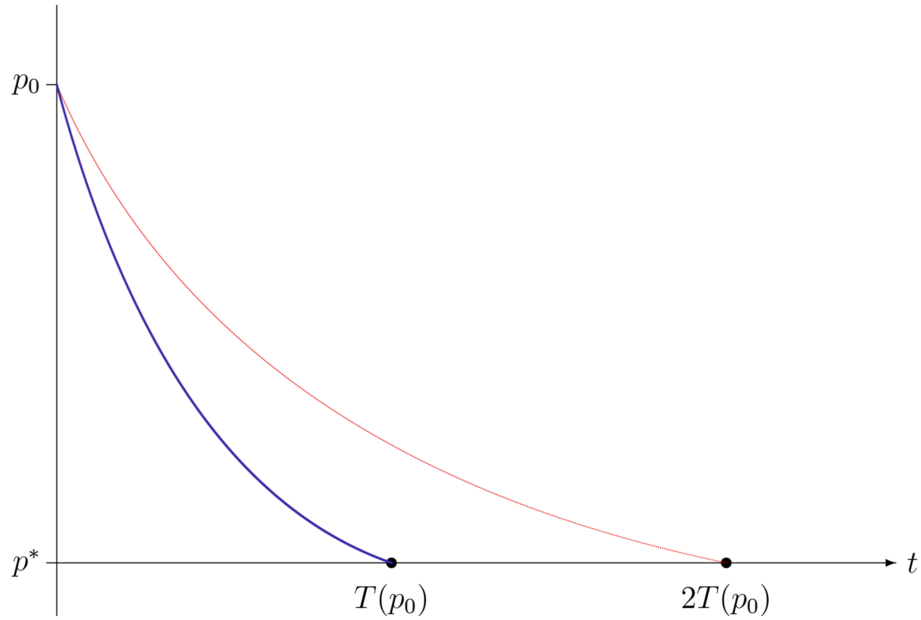

Figure 1: Belief Decay

When two firms are active, beliefs decay twice as fast (lower curve) as with one firm (upper curve).

The optimal quitting time for a firm with signal s_i is just $2T(s_i)$ since from the definition of T in (3) and (4),

$$e^{-2\lambda T(s_i)} \frac{p(s_i)}{1-p(s_i)} = \frac{p^*}{1-p^*} \tag{7}$$

Since the beliefs of a single firm decay at one-half the rate of decay with two firms—two failures constitute worse news than one failure—it takes twice as long to reach p^*, as depicted in Figure 1. Since $2T(s_i)$ is the single-firm optimal quitting time, using (7), the probability of success given the initial belief $p(s_i)$ is

$$p(s_i)\left(1-e^{-2\lambda T(s_i)}\right) = \frac{p(s_i)-p^*}{1-p^*}$$

Consider firm i when alone. If $p^* < p(b_i) < p(g_i)$, the firm would enter regardless of its signal and its ex ante probability of success is

$$\begin{aligned} \Pr[\mathcal{S}_i] &= \Pr[g_i]\frac{p(g_i)-p^*}{1-p^*} + \Pr[b_i]\frac{p(b_i)-p^*}{1-p^*} \\ &= \frac{\pi - p^*}{1-p^*} \end{aligned}$$

If $p(b_i) \leq p^* < p(g_i)$, the firm would enter only if its signal were g_i and now the ex ante probability of success is

$$\begin{aligned}
\Pr[\mathcal{S}_i] &= \Pr[g_i] \frac{p(g_i) - p^*}{1 - p^*} + \Pr[b_i] \times 0 \\
&= \frac{\pi q_i - (\pi q_i + (1-\pi)(1-q_i)) p^*}{1 - p^*} \\
&= \frac{\pi q_i (1 - p^*) - (1-\pi)(1-q_i) p^*}{1 - p^*}
\end{aligned}$$

The proof of Proposition 0 is divided into four cases.

Case 1: $p^* \leq p(b_1) < p(b_2)$ In this case, both firms would enter regardless of their signals and so

$$\Pr[\mathcal{S}_1] = \frac{\pi - p^*}{1 - p^*} = \Pr[\mathcal{S}_2]$$

Thus, the two firms would have the same probability of success and the same expected payoffs.

Case 2: $p(b_1) < p^* < p(b_2)$ Now firm 1 would enter only with a good signal whereas firm 2 would enter regardless of its signal. Thus,

$$\begin{aligned}
\Pr[\mathcal{S}_1] - \Pr[\mathcal{S}_2] &= \frac{\pi q_1 (1 - p^*) - (1-\pi)(1-q_1) p^*}{1 - p^*} - \frac{\pi - p^*}{1 - p^*} \\
&= \frac{(1-\pi) q_1 p^* - \pi (1-q_1)(1-p^*)}{1 - p^*} \\
&= (1-\pi) q_1 \left(\frac{p^*}{1-p^*} - \frac{\pi (1-q_1)}{(1-\pi) q_1} \right) \\
&= (1-\pi) q_1 \left(\frac{p^*}{1-p^*} - \frac{p(b_1)}{1 - p(b_1)} \right) \\
&> 0
\end{aligned}$$

Case 3: $p(b_2) \leq p^* < p(g_2)$ In this case, both firms would enter only if they had good signals and some routine calculations show that the difference in success probabilities

$$\begin{aligned}
\Pr[\mathcal{S}_1] - \Pr[\mathcal{S}_2] &= \frac{\pi (1-p^*) + (1-\pi) p^*}{1 - p^*} (q_1 - q_2) \\
&> 0
\end{aligned}$$

Case 4: $p(b_2) < p(g_2) \leq p^*$ In this case, firm 2 would not enter regardless of its signal and so its probability of success is zero.

This completes the proof of Proposition 0. ■

4 Upstart equilibrium

In this section, we exhibit an equilibrium of the R&D game in which the probability that the startup wins the race is *greater* than or equal to the probability that the established firm wins and is strictly greater whenever $p(b_1, g_2) > p^*$. The expected payoff if the startup is also greater than or equal to the payoff of the established firm.

In the next section, we will show that when the established firm 1 is much better informed than the startup firm 2, this is the *unique* Nash equilibrium outcome.

Recall from (3) and (5) that if $p(s_1, s_2) = \Pr[G \mid s_1, s_2] > p^*$, the two-firm threshold time $T(s_1, s_2)$ is defined by

$$e^{-2\lambda T(s_1,s_2)} \frac{p(s_1, s_2)}{1 - p(s_1, s_2)} = \frac{p^*}{1 - p^*} \tag{8}$$

and if $p(s_1, s_2) \leq p^*$, then $T(s_1, s_2) = 0$.

The ranking of the posterior probabilities (see (1)) implies

$$T(b_1, b_2) \leq T(b_1, g_2) \leq T(g_1, b_2) \leq T(g_1, g_2) \tag{9}$$

and the inequalities are strict unless both sides are 0.

Consider following "upstart outcome" depicted in Figure 2. When the signals are (b_1, b_2), firm 1 exits early at $T(b_1)$, as defined in (4), and firm 2 exits immediately afterwards.[11] When the signals are (g_1, g_2), both firms exit simultaneously at time $T(g_1, g_2)$. When the signals are (g_1, b_2), firm 2 exits at time $T(g_1, b_2)$ and firm 1 follows immediately. Finally, when the signals are (b_1, g_2), firm 1 exits at $T(b_1)$ and firm 2 exits at $2T(b_1, g_2) - T(b_1)$.

In the first three cases the chance that firm 1 will win is the same as the chance that firm 2 will win. But in the last case, firm 1 exits at $T(b_1)$ and as long as $T(b_1, g_2) > T(b_1)$, firm 2 has a greater chance of winning. Thus, ex ante firm 2 has a greater chance of obtaining the patent than does firm 1—the startup is an upstart. We will first establish

Proposition 1 *There exists a perfect Bayesian equilibrium in which the less-informed firm* 2 *wins more often, and has a higher payoff, than the better-informed firm* 1*.*

Strategies A *strategy* for firm i is a pair of functions (τ_i, σ_i) where $\tau_i : \{g_i, b_i\} \to \mathbb{R}_+ \cup \{\infty\}$ and $\sigma_i : \{g_i, b_i\} \times \mathbb{R}_+ \to \mathbb{R}_+ \cup \{\infty\}$.

First, $\tau_i(s_i)$ is the time at which firm i with signal s_i decides to quit unilaterally—that is, if he or she has not received any information that the other firm has quit. If $\tau_i(s_i) = \infty$, this means that the firm decides to never quit unilaterally.

Second, $\sigma_i(s_i, t_j)$ is the time at which firm i with signal s_i quits after learning that the other firm quit at time t_j. Of course, $\sigma_i(s_i, t_j) \geq t_j$.

[11] It could be, of course, that $T(b_1) = 0$ and in that case exiting at $T(b_1)$ is the same as not entering.

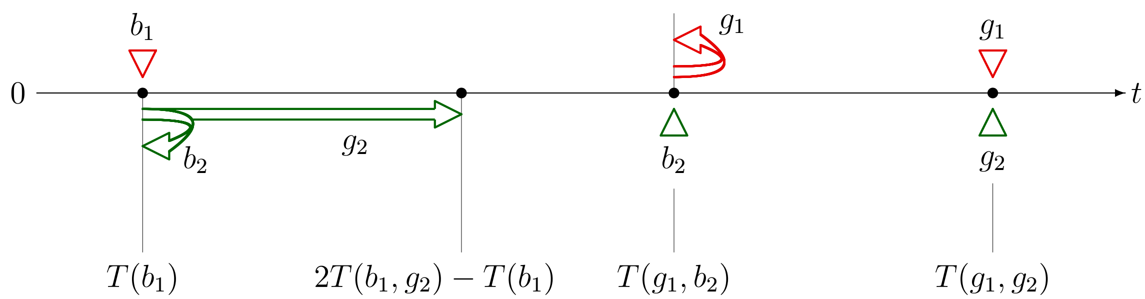

Figure 2: Upstart Equilibrium

Firm 1 (top) with signal b_1 exits at $T(b_1)$. The exit decision of firm 1 with g_1 depends on firm 2. If firm 2 exits at $T(g_1, b_2)$, then firm 1 with signal g_1 follows immediately, depicted as a U-turn. Otherwise, it stays until $T(g_1, g_2)$. Similarly, the exit decisions of firm 2 (bottom) depend on firm 1. If firm 1 exits at $T(b_1)$, firm 2 with signal b_2 follows immediately and with g_2, exits at $2T(b_1, g_2) - T(b_1)$. Otherwise, firm 2 with b_2 exits at $T(g_1, b_2)$ and with g_2, exits at $T(g_1, g_2)$.

Since this is a game in continuous time it is worthwhile to explicitly state how strategies translate into outcomes. If the signals are (s_i, s_j), and $\tau_i(s_i) < \tau_j(s_j)$, then of course firm i exits at $\tau_i(s_i)$ and firm j at $\sigma_j(s_j, \tau_i(s_i))$. If $\tau_i(s_i) = \tau_j(s_j)$, then both firms exit simultaneously.

We have only defined pure strategies here as the equilibrium we construct below does not involve any randomization. When we show that the equilibrium outcome is unique, we will introduce and consider randomized strategies as well.

4.1 Equilibrium strategies

Consider the following strategies:

Firm 1 :

$$\tau_1^*(g_1) = T(g_1, g_2) \text{ and } \tau_1^*(b_1) = T(b_1)$$

$$\sigma_1^*(g_1, t_2) = \begin{cases} T(g_1, b_2) & \text{if } t_2 = T(g_1, b_2) \\ 2T(g_1, b_2) - t_2 & \text{if } t_2 < T(g_1, b_2) \\ 2T(g_1, g_2) - t_2 & \text{if } T(g_1, b_2) < t_2 < T(g_1, g_2) \end{cases}$$

with the following beliefs about its rival. If $t_2 \leq T(g_1, b_2)$, then firm 1 believes that $s_2 = b_2$ and otherwise believes $s_2 = g_2$.

Firm **2:**

$$\tau_2^*(g_2) = T(g_1, g_2) \text{ and } \tau_2^*(b_2) = T(g_1, b_2)$$

$$\sigma_2^*(g_2, t_1) = \begin{cases} 2T(b_1, g_2) - t_1 & \text{if } t_1 \le T(b_1) \\ 2T(g_1, g_2) - t_1 & \text{if } T(b_1) < t_1 < T(g_1, g_2) \end{cases}$$
$$\sigma_2^*(b_2, t_1) = \begin{cases} t_1 & \text{if } t_1 \le T(b_1) \\ 2T(g_1, b_2) - t_1 & \text{if } T(b_1) < t_1 < T(g_1, b_2) \end{cases}$$

with the following beliefs about its rival. If $t_1 \le T(b_1)$, then firm 2 believes that $s_1 = b_1$ and otherwise believes $s_1 = g_1$.

4.2 Verification of equilibrium

We now verify that the strategies (τ_i^*, σ_i^*) specified above constitute a perfect Bayesian equilibrium. To do this we will ascertain the optimal quitting time for the two firms in various situations. This quitting time will, as in Lemma 3.1, be determined by the condition that a firm's belief that the state is G is equal to p^*. But when another firm j is present, firm i not only knows its own signal s_i but may learn firm j's signal s_j in the course of play. Thus, it may be the case that even if based on its own signal alone, the belief is below p^*, the possibility of learning s_j in the future is a worthwhile investment. The following analog of Lemma 3.1 is derived under the condition that all such learning has already taken place. Thus we have

Lemma 4.1 *Let p_{it} denote i's belief at time t that the state is G.*
(i) If $p_{it} > p^*$, then i should not quit at t.
(ii) Suppose that at time t firm i believes with probability one that j's signal is s_j. If $p_{it} \le p^*$, then firm i should quit at t.

Proof. The flow profit of firm i if it quits at time t_i is

$$r \int_0^{t_i} e^{-rt} \Pr[\mathcal{S}_0(t)] \left(p_{it} \frac{\lambda m}{r} - c \right) dt = \lambda m \int_0^{t_i} e^{-rt} \Pr[\mathcal{S}_0(t)] (p_{it} - p^*) \, dt$$

where p_{it} is firm i's belief at time t given all the information it has and $\Pr[\mathcal{S}_0(t)]$ is the probability that there has been no success until time t. This is the payoff because the chance that both firms will succeed at the same instant is zero. Note that firm j's quitting time t_j affects the instantaneous payoff only through its effect on i's belief p_{it}—before t_j the belief p_{it} declines rapidly since there are two unsuccessful firms whereas after j quits at time t_j the belief declines slowly since there is only one unsuccessful firm. ■

Firm 1 Suppose firm 2 follows the strategy (σ_2^*, τ_2^*) specified above.

Firm 1 **with signal** g_1 : We first argue that $\tau_1(g_1) < T(g_1, b_2)$ cannot be a best response. This is because $\tau_2^*(b_2) = T(g_1, b_2) < T(g_1, g_2) = \tau_2^*(g_2)$ and if g_1 exits before $T(g_1, b_2)$, it cannot learn 2's signal and the only information it has until then is g_1. But the posterior probability of G conditional on g_1 alone is $p(g_1) > p(g_1, b_2)$. And if there has been no success until $t < T(g_1, b_2)$, 1's belief $p_{1t} = p_t(g_1) > p_t(g_1, b_2)$ for $t < T(g_1, b_2)$. By Lemma 4.1, it is suboptimal to quit before $T(g_1, b_2)$.

On the other hand, if $\tau_1(g_1) \geq T(g_1, b_2)$, there are two possibilities. Given τ_2^*, either g_1 learns at time $T(g_1, b_2)$ that firm 2 quit and then infers that $s_2 = b_2$ or g_1 learns that firm 2 did not quit and then infers that $s_2 = g_2$. If g_1 learns that 2 quit, then it should also quit as soon as possible, that is, at $T(g_1, b_2)$ (Lemma 4.1 again). Thus, $\sigma_1^*(g_1, T(g_1, b_2))$ is optimal. If g_1 learns that 2 did not quit at time $T(g_1, b_2)$, then since $\tau_2^*(g_2) = T(g_1, g_2)$, firm 1 should exit at $T(g_1, g_2)$ as well, that is, $\tau_1^*(g_1)$ is optimal. It is obvious that firm 1's beliefs about s_2 are consistent with firm 2's equilibrium behavior.

By the same reasoning, $\sigma_1^*(g_1, t_2)$ is optimal for all $t_2 \neq T(g_1, b_2)$ given 1's (off-equilibrium) beliefs.

Firm 1 with signal b_1 : We will argue that given (τ_2^*, σ_2^*) it is optimal for b_1 to exit at $T(b_1)$. Depending on its signal, firm 2 will quit at either $\tau_2^*(b_2) = T(g_1, b_2)$ or at $\tau_2^*(g_2) = T(g_1, g_2)$. First, $T(b_1) < \tau_1(b_1) < T(g_1, b_2)$ is not optimal because 1 will not learn anything about 2's signal. If firm 1 chooses $\tau_1(b_1) \geq T(g_1, b_2)$ and finds that 2 is still active, then it believes that firm 2's signal is g_2 and that the time when firm 2 will quit is $T(g_1, g_2)$. But now by Lemma 4.1 it is best to quit at $T(b_1, g_2) \leq T(g_1, b_2)$, the time when 1 can learn 2's signal. This means that the value of staying and learning 2's signal at $T(g_1, b_2)$ is negative. Thus, $\tau_1^*(b_1) = T(b_1)$ is optimal.

Firm 2 Now suppose firm 1 follows the strategy (σ_1^*, τ_1^*) specified above and consider

Firm 2 with signal g_2 : First, since $\tau_1^*(b_1) = T(b_1)$, it is not optimal for firm 2 to quit before $T(b_1)$ either. Moreover, since $\tau_1^*(g_1) = T(g_1, g_2)$, if firm 2 stays until $T(b_1)$, it will learn whether 1's signal is b_1 or g_1. If it learns that $s_1 = b_1$, then its optimal response is $\sigma_2^*(g_2, T(b_1)) = 2T(b_1, g_2) - T(b_1)$. This is because once it knows that $s_1 = b_1$, its belief at time $2T(b_1, g_2) - T(b_1)$ is such that

$$\frac{p_{2t}}{1 - p_{2t}} = \frac{p(b_1, g_2)}{1 - p(b_1, g_2)} e^{-\lambda T(b_1)} e^{-\lambda(2T(b_1, g_2) - T(b_1))}$$

and this is, by definition, equal to $p^* / (1 - p^*)$ if $T(b_1, g_2) > 0$ and no greater than $p^* / (1 - p^*)$ if $T(b_1, g_2) = 0$. On the other hand, if it learns that $s_1 = g_1$, then by Lemma 4.1 firm 2 should quit at $\tau_2^*(g_2) = T(g_1, g_2)$.

By the same reasoning, $\sigma_2^*(g_2, t_1)$ is optimal for all $t_1 < T(g_1, g_2)$ given 2's (off-equilibrium) beliefs.

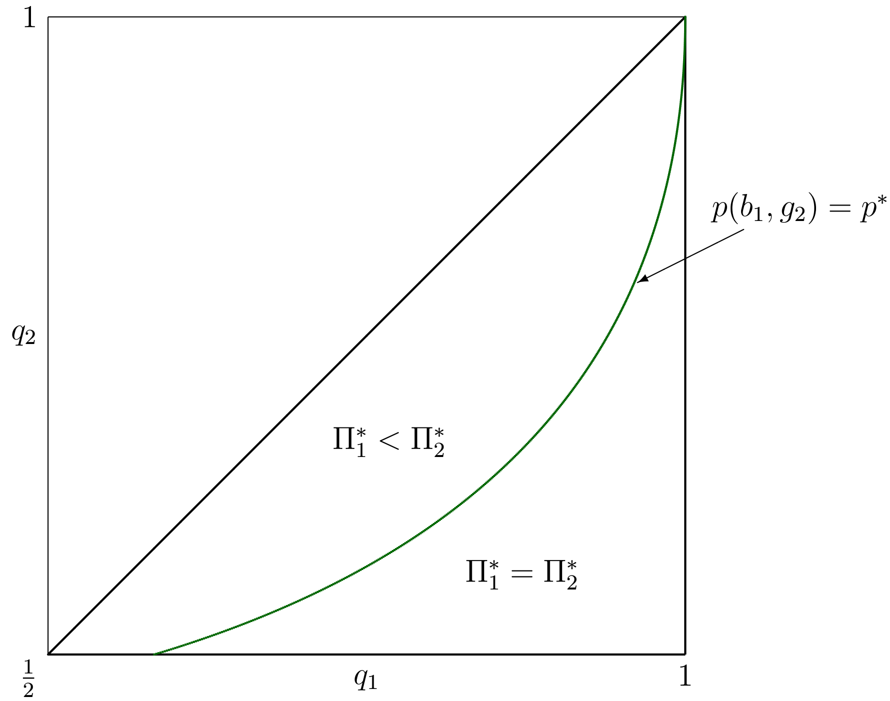

Figure 3: Upstart Equilibrium Payoffs

An upstart equilibrium exists for all $q_1 > q_2$. Above the curve, firm 2's equilibrium payoff (and winning probability) is strictly greater than that of firm 1. Below the curve, they are equal.

Firm 2 **with signal** b_2 : The same reasoning as in the case where firm 2's signal was g_2 shows that again 2's strategy is a best response.

This completes the proof of Proposition 1. ■

The particular choice of off-equilibrium beliefs does not affect the equilibrium outcome—any beliefs will do. Off-equilibrium beliefs could affect a firm's profit only if a deviation to stay longer than expected would cause its rival to drop out earlier. For instance, the equilibrium specifies that firm 1 with signal b_1 should exit at $T(b_1)$. If it did not, then firm 2 would have to assign probability 1 to $s_1 = g_1$, since this is the only belief consistent with the equilibrium strategy. Thus, by staying longer that $T(b_1)$ firm 1 cannot get firm 2 to exit early. In the upstart equilibrium outcome, all such events occur on the equilibrium path.

4.3 Equilibrium payoffs

The expected flow profits of firm 1 in the upstart equilibrium are

$$\begin{aligned}\Pi_1^* &= \Pr[g_1, g_2] \times v(p(g_1, g_2)) + \Pr[g_1, b_2] \times v(p(g_1, b_2)) \\ &\quad + \Pr[b_1] \times v(p(b_1))\end{aligned} \tag{10}$$

where $v(p_0)$ is the flow payoff to a firm when both firms have a common belief p_0 at time 0 that the state is G (see Appendix A). The expression for the equilibrium payoff results from the fact that when the signals are (g_1, g_2) or (g_1, b_2), these become commonly known in the course of play of the upstart equilibrium. When its signal is b_1, firm 1 acquires no additional information about 2's signal and exits at $T(b_1)$.

The expected profits of firm 2 in the upstart equilibrium are

$$\begin{aligned}\Pi_2^* &= \Pr[g_1, g_2] \times v(p(g_1, g_2)) + \Pr[g_1, b_2] \times v(p(g_1, b_2)) \\ &\quad + \Pr[b_1] \times v(p(b_1)) + e^{-rT(b_1)} \Pr[b_1, g_2] \times u\left(p_{T(b_1)}(b_1, g_2)\right) \end{aligned} \tag{11}$$

where $u(p_0)$ is the flow payoff to a firm when it is alone with belief p_0 at time 0 (see Appendix A again). To see why this is firm 2's payoff note that when the signals are (g_1, g_2) or (g_1, b_2) firm 2 exits at the same time as firm 1. Now if firm 1's signal is b_1, the two firms receive the same payoff until time $T(b_1)$ when firm 1 exits. When firm 2's signal is b_2, then it too exits at $T(b_1)$ and so receives no additional payoff. When firm 2's signal is g_2, however, it continues to stay longer—until its belief erodes to p^*. The additional flow payoff from staying beyond $T(b_1)$ is the single-firm value function u evaluated at the belief that firm 2 has at time $T(b_1)$. This belief is $p_{T(b_1)}(b_1, g_2)$ and the consequent payoff is discounted back to time 0.

Thus,

$$\Pi_2^* - \Pi_1^* = e^{-rT(b_1)} \Pr[b_1, g_2] \times u\left(p_{T(b_1)}(b_1, g_2)\right) \tag{12}$$

and this is strictly positive if $T(b_1, g_2) > 0$ or equivalently, if $p(b_1, g_2) > p^*$. Note that a necessary condition for this is that $\pi > p^*$.

These facts are depicted in Figure 3 and we summarize these findings as,

Proposition 2 *In the upstart equilibrium, the less-informed firm 2's payoff is no less than the better-informed firm 1's payoff and strictly greater if $p(b_1, g_2) > p^*$.*

Corollary 1 *If $q_1 = 1$, then the payoffs of the two firms are equal. Also, if $q_2 = \frac{1}{2}$, the payoffs of the two firms are equal as well.*

Proof. If $q_1 = 1$, then firm 1's signal is perfectly informative about the state and so $p(b_1, g_2) = 0$. Thus, it is also the case that $p_{T(b_1)}(b_1, g_2) = 0$ and since $u(0) = 0$, from (12), the payoffs of the two firms are equal.

If $q_2 = \frac{1}{2}$, then $p(b_1, g_2) = p(b_1)$. This means that $p_{T(b_1)}(b_1, g_2) = p_{T(b_1)}(b_1) \leq p^*$ and so again $u\left(p_{T(b_1)}(b_1, g_2)\right) = 0$. Now from (12) again, the payoffs of the two firms are equal. ■

5 Uniqueness

We now show that when the informational advantage of firm 1 is large, that is, fixing all other parameters, q_2 is small relative to q_1, then the upstart equilibrium outcome is the unique Nash equilibrium outcome.

Proposition 3 *When the established firm's informational advantage is large, there is a unique Nash equilibrium outcome. Precisely, for every q_1 there exists a q_2^+ such that for all $q_2 < q_2^+$, there is a unique Nash equilibrium outcome.*

The proof of the Proposition is in two steps. First, we show that iterated elimination of dominated strategies (IEDS) results in a *single* outcome. Here we will use one round of elimination of *weakly* dominated strategies, followed by multiple (actually six more!) rounds of (iteratively) *weakly/strictly* dominated strategies.[12] The resulting outcome will be the same as in (τ^*, σ^*). As a final step, we will show that there cannot be any other Nash equilibrium outcome—the weakly dominated strategies that were eliminated cannot be part of any Nash equilibrium.

The formal argument is somewhat involved but the key steps are easily understood. Let us consider firm 1's unilateral exit times $\tau_1(b_1)$ and $\tau_1(g_1)$. It is clear that firm 1 with signal g_1 should stay at least until $T(g_1, b_2)$ which is the optimal exit time under the assumption that the rival's information is as bad as possible. Similarly, it is clear that firm 1 with signal b_1 should stay no longer than $T(b_1, g_2)$ which is the optimal exit time under the assumption that the rival's information is as good as possible. Since $T(b_1, g_2) < T(g_1, b_2)$, we have $\tau_1(b_1) \leq T(b_1, g_2) < T(g_1, b_2) \leq \tau_1(g_1)$. In other words, firm 1's strategy fully reveals its information at the latest by time $T(b_1, g_2)$. Thus firm 2 can learn firm 1's signal for sure by investing until $T(b_1, g_2)$. When the quality of firm 2's information is small relative to q_1, a simple calculation shows that investing to learn firm 1's information is worthwhile for firm 2. The remaining steps in the proof are rather straightforward.

The arguments below show not only that the upstart outcome is the unique Nash outcome but also that the upstart outcome is "almost" the unique rationalizable outcome. Precisely, as argued below it is the unique outcome of iterated conditional dominance.

5.1 Step 1

Denote by Γ the original game and by $\Gamma(n)$ the game after n rounds of elimination. In what follows, Lemma 4.1 will invoked repeatedly in the following manner: if the two signals are known to be (s_1, s_2), then a firm that exits at $t < T(s_1, s_2)$ would leave some money on the table since that firm's belief time t, $p_{it} > p^*$.

IEDS Round 1

Claim 1 (a) *Any strategy of firm 1 such that $\tau_1(g_1) < T(g_1, b_2)$ is **weakly** dominated in Γ.*

[12] The weakly dominated strategies that we eliminate are so only because of histories that never occur. Below we will show that the upstart outcome is also the result of iterated elimination of *conditionally dominated* strategies.

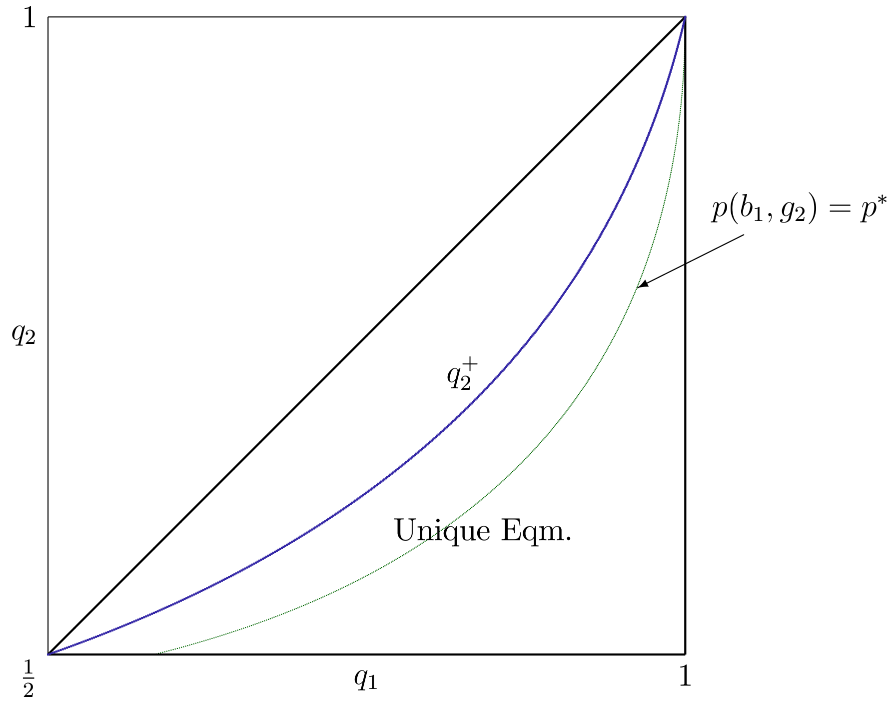

Figure 4: Uniqueness

There is a unique equilibrium outcome below the upper curve. Between the two curves, the startup has strictly higher payoffs in the unique equilibrium outcome.

Proof. Quitting at $\tau_1(g_1) < T(g_1, b_2)$ is weakly dominated by quitting at $\overline{\tau}_1(g_1) = T(g_1, b_2)$. First, if $\max(\tau_2(b_2), \tau_2(g_2)) \geq T(g_1, b_2)$ then quitting at $\tau_1(g_1) < T(g_1, b_2)$ is strictly worse for g_1 than quitting at $T(g_1, b_2)$. If $\max(\tau_2(b_2), \tau_2(g_2)) < T(g_1, b_2)$, then quitting at $\tau_1(g_1) < T(g_1, b_2)$ is strictly worse than quitting at $T(g_1, b_2)$ if $\tau_1(g_1) < \max(\tau_2(b_2), \tau_2(g_2))$ and is equivalent if $\max(\tau_2(b_2), \tau_2(g_2)) < \tau_1(g_1)$. ■

Claim 1 (b) *Any strategy of firm 2 such that $\tau_2(g_2) < T(b_1, g_2)$ is **weakly** dominated in Γ.*

Proof. The proof is the same as in the previous claim with the identities the firms interchanged. ■

It is important to note that in this round the strategies eliminated are not *strictly* dominated. The reason is that a strategy (τ_1, σ_1) that calls on firm 1 with signal g_1 to quit at a time such that $0 < \tau_1(g_1) < T(g_1, b_2)$ is not strictly worse than quitting at $T(g_1, b_2)$ against a strategy (τ_2, σ_2) such that $\tau_2(b_2) = 0 = \tau_2(g_2)$. Since both types of firm 2 quit at time 0, the choice of $\tau_1(g_1)$ is irrelevant. More generally, such a $\tau_1(g_1)$ is not strictly worse than $T(g_1, b_2)$ against any strategy (τ_2, σ_2) such that $\max(\tau_2(b_2), \tau_2(g_2)) < \tau_1(g_1)$.

IEDS Round 2

Claim 2 (a) *Any strategy of firm* 1 *such that* $\tau_1(b_1) > T(b_1, g_2)$ *is strictly dominated in* $\Gamma(1)$.

Proof. If firm 2's signal is g_2, then from Claim 1 (b) $\tau_2(g_2) \geq T(b_1, g_2)$. In this case, for firm 1 to choose $\tau_1(b_1) > T(b_1, g_2)$ is strictly worse than $\tau_1(b_1) = T(b_1, g_2)$. On the other hand, if firm 2's signal is b_2, then for firm 1 to choose $\tau_1(b_1) > T(b_1, g_2)$ is no better than $\tau_1(b_1) = T(b_1, g_2)$. Thus, the expected payoff from $\tau_1(b_1) > T(b_1, g_2)$ is strictly lower than the expected payoff from quitting at $T(b_1, g_2)$. ■

Claim 2 (b) *Any strategy of firm* 2 *such that for* $s_2 = b_2$ *or* g_2, $\tau_2(s_2) < T(b_1, b_2)$ *is strictly dominated in* $\Gamma(1)$.

Proof. Clearly, the worst possible case for firm 2 is if firm 1 has the signal b_1. For firm 2 with either signal exit before $T(b_1, b_2)$ is no better than staying until $T(b_1, b_2)$. But if firm 1's signal is g_1, then from Claim 1 (a) $\tau_1(g_1) \geq T(g_1, b_2) \geq T(b_1, b_2)$. Since with positive probability firm 1 will stay until $T(b_1, b_2)$, it is strictly dominated for firm 2 to quit before then. ■

IEDS Round 3

Claim 3 *Given all other parameters, there exists a* q_2^+ *such that for all* $q_2 < q_2^+$, *any strategy of firm 2 such that (i) if* $T(g_1, b_2) > 0$, *then* $\tau_2(b_2) < T(b_1, g_2)$ *is strictly dominated in* $\Gamma(2)$; *and (ii) if* $T(g_1, b_2) = 0$, *then* $\tau_2(b_2) > 0$ *is strictly dominated in* $\Gamma(2)$.

Proof. (i) Claim 1 (a) and Claim 2 (a) imply that $\tau_1(b_1) \leq T(b_1, g_2) < T(g_1, b_2) \leq \tau_1(g_1)$. This means that firm 2 can learn firm 1's signal by staying until $\tau_1(b_1)$.

We will now argue that if $\tau_2(b_2) < T(b_1, g_2)$ then (τ_2, σ_2) is strictly dominated by $(\overline{\tau}_2, \overline{\sigma}_2)$ such that $\overline{\tau}_2(b_2) = T(g_1, b_2)$ and $\overline{\sigma}_2(b_2, t_1) = t_1$ for all $t_1 \leq T(b_1, g_2)$. From Claim 2 (b) we already know that to exit before $T(b_1, b_2)$ is strictly dominated for b_2. Thus, it is enough to establish that it is also strictly dominated to exit at any time T between $T(b_1, b_2)$ and $T(g_1, b_2)$. Since b_1 will exit no later than $T(b_1, g_2)$, firm 2's flow profit from the strategy $(\overline{\tau}_2, \overline{\sigma}_2)$ when evaluated at time T is at least

$$\lambda m \int_T^{T(b_1, g_2)} e^{-r(t-T)} \Pr[\mathcal{S}_0(t)] (p_{2t} - p^*) \, dt$$
$$+ \Pr[g_1 \mid b_2, \mathcal{S}_0(T)] \times \lambda m \int_{T(b_1, g_2)}^{T(g_1, b_2)} e^{-r(t-T)} \Pr[\mathcal{S}_0(t)] (p_{2t} - p^*) \, dt$$

where $\mathcal{S}_0(t)$ is the event that neither firm has succeed until t and firm 2's belief at time t that the state is G is

$$\frac{p_{2t}}{1 - p_{2t}} = \begin{cases} e^{-2\lambda t} \frac{p(b_2)}{1 - p(b_2)} & \text{if } t \leq T(b_1, g_2) \\ e^{-2\lambda t} \frac{p(g_1, b_2)}{1 - p(g_1, b_2)} & \text{if } t > T(b_1, g_2) \end{cases}$$

Before time $T(b_1, g_2)$, firm 2 cannot learn 1's signal and so its belief p_{2t} results only from its own signal b_2. At time $T(b_1, g_2)$ it learns 1's signal and exits immediately if $s_1 = b_1$. But if firm 1 does not exit at $T(b_1, g_2)$, then firm 2 knows that $s_1 = g_1$ and its belief now results from both its own signal b_2 and firm 1's signal g_1.

Notice that while the first term in the expression for firm 2's payoff above may be negative, the second is surely positive. As $q_2 \downarrow \frac{1}{2}$, $p(b_1, g_2) \downarrow p(b_1, b_2)$, or equivalently, $T(b_1, g_2) \downarrow T(b_1, b_2)$, and since $T \in [T(b_1, b_2), T(b_1, g_2)]$ the first term approaches zero while the second is strictly positive when $T(g_1, b_2) > 0$. Thus, there exists a $q_2^+ > \frac{1}{2}$ such that for all $q_2 < q_2^+$, the payoff from $(\overline{\tau}_2, \overline{\sigma}_2)$ is greater than the payoff from any strategy such that $\tau_2(b_2) < T(b_1, g_2)$.

If $s_1 = b_1$, then 2 is indifferent at all $\tau_2(b_2) > T(b_1, g_2)$. But if $s_1 = g_1$, $\overline{\tau}_2(b_2) = T(g_1, b_2)$ is strictly better than $\tau_2(b_2) < T(b_1, g_2)$. Since the latter occurs with positive probability, $(\overline{\tau}_2, \overline{\sigma}_2)$ is *strictly* better.

(ii) Obvious since in this case $p(g_1, b_2) \leq p^*$. ■

In the rest of the proof, we will assume that $q_2 < q_2^+$. Note that q_2^+ depends on the other parameters, in particular on q_1.

IEDS Round 4

Claim 4 *Any strategy of firm* 1 *such that* $\tau_1(b_1) \neq T(b_1)$*, is strictly dominated in* $\Gamma(3)$.

Proof. We will first argue that $\tau_1(b_1) > T(b_1)$ is strictly dominated. From Claim 1 (b) and Claim 3 we know that firm 2, regardless of its signal, will not be the first to quit before $T(b_1, g_2)$. This means that firm 1 will learn nothing from firm 2 prior to $T(b_1, g_2)$. This implies that if firm 1 with signal b_1 exits at any time $t_1 \in (T(b_1), T(b_1, g_2)]$, its flow payoff after $T(b_1)$ is negative (recall that $T(b_1)$ is the optimal exit time for firm 1 with only his own signal b_1). If firm 1 stays until $T(b_1, g_2)$ or longer, the best event is that it learns that firm 2's signal is g_2 at exactly time $T(b_1, g_2)$, the earliest time that he could learn anything about firm 2's signal. But even in this case, it is best to exit immediately after learning firm 2's signal. Thus, even if firm 1 were to learn that firm 2's signal was g_2, it cannot make any use of this information. Thus, staying after $T(b_1)$ is strictly dominated for b_1.

Clearly there is no reason for b_1 to quit before $T(b_1)$. ■

IEDS Round 5

Claim 5 (a) *Any strategy of firm* 2 *such that* $\sigma_2(g_2, T(b_1)) \neq 2T(b_1, g_2) - T(b_1)$ *is strictly dominated in* $\Gamma(4)$.

Proof. Given all previous rounds, we know that firm 1 with b_1 will exit at $T(b_1)$ and with signal g_1 will exit no earlier than $T(b_1, g_2) > T(b_1)$. Thus, if firm 2 sees at time $T(b_1)$ that firm 1 exits, it knows that 1's signal was b_1. If firm 2's signal is g_2, it is now strictly dominated to quit at a time other than $2T(b_1, g_2) - T(b_1)$, the time when g_2's beliefs will reach p^*. ■

Claim 5 (b) *Any strategy of firm* 2 *such that* $\sigma_2(b_2, T(b_1)) \neq T(b_1)$ *is strictly dominated in* $\Gamma(4)$.

Proof. Given all previous rounds, we know that firm 1 with b_1 will exit at $T(b_1)$ and with g_1 will stay longer. Thus, if firm 2 sees at time $T(b_1)$ that firm 1 exits, it knows that 1's signal was b_1. Clearly, given that 2's own signal is b_2, staying any longer is strictly dominated. ■

Claim 5 (c) *Any strategy of firm* 2 *such that* $\tau_2(b_2) \neq T(g_1, b_2)$ *is strictly dominated in* $\Gamma(4)$.

Proof. Given all previous rounds, we know that firm 1 with b_1 will exit at $T(b_1)$ and with g_1 will stay longer. Thus, if firm 2 sees at time $T(b_1)$ that firm 1 did not exit, it knows that 1's signal is g_1. From Claim 1(a), firm 1 will stay at least until $T(g_1, b_2)$. For firm 2 with signal b_2 to quit at a time other than $T(g_1, b_2)$ is strictly dominated. ■

Claim 5 (d) *Any strategy of firm* 2 *such that* $\tau_2(g_2) < T(g_1, g_2)$ *is* ***weakly*** *dominated in* $\Gamma(4)$.

Proof. Given all previous rounds, we know that firm 1 with b_1 will exit at $T(b_1)$ and with g_1 will stay longer. Thus, if firm 2 sees at time $T(b_1)$ that firm 1 did not exit, it knows that 1's signal is g_1. If $\tau_1(g_1) \geq T(g_1, g_2)$, then $\tau_2(g_2) < T(g_1, g_2)$ is strictly worse than quitting at $T(g_1, g_2)$. If $\tau_1(g_1) < T(g_1, g_2)$, then all quitting times $\tau_2(g_2)$ such that $\tau_1(g_1) < \tau_2(g_2)$ result in the *same* payoff as quitting at $T(g_1, g_2)$. If $\tau_1(g_1) < T(g_1, g_2)$, then all quitting times $\tau_2(g_2)$ such that $\tau_2(g_2) < \tau_1(g_1)$ results in a payoff strictly worse than from quitting at $T(g_1, g_2)$. ■

Note that for the same reasons as in Round 1, the strategies eliminated in Claim 5 (d) are also only weakly dominated.

IEDS Round 6

Claim 6 (a) *Any strategy of firm* 1 *such that* $\sigma_1(g_1, T(g_1, b_2)) \neq T(g_1, b_2)$ *is strictly dominated in* $\Gamma(5)$

Proof. Given all previous rounds, $\tau_2(b_2) = T(g_1, b_2) < T(g_1, g_2) \leq \tau_2(g_2)$ (Claim 5 (c) and Claim 5 (d)). So if firm 2 quits at $T(g_1, b_2)$, firm 1 knows that 2's signal is b_2. Then it is dominated for firm 1 to continue after $T(g_1, b_2)$. ■

Claim 6 (b) *Any strategy of firm* 1 *such that* $\tau_1(g_1) \neq T(g_1, g_2)$ *is strictly dominated in* $\Gamma(5)$.

Proof. As in the proof of the previous claim, if firm 2 does not quit at $T(g_1, b_2)$, firm 1 knows that 2's signal is g_2. From Claim 5(d), $\tau_2(g_2) \geq T(g_1, g_2)$. Thus, it is dominated for firm 1 to quit at any other time. ■

IEDS Round 7

Claim 7 *Any strategy of firm* 2 *such that* $\tau_2(g_2) > T(g_1, g_2)$ *is strictly dominated in* $\Gamma(6)$.

Proof. If firm 2 with signal g_2 sees that firm 1 stayed beyond $T(b_1)$, it knows that 1's signal is g_1. From Claim 6 (b), thus firm 1 will quit at $T(g_1, g_2)$ and so firm 2 should also quit at that time. ■

5.2 Step 2

The iterated elimination of dominated strategies, weak and strict, carried out above leaves a single *outcome*—the same as that in the upstart equilibrium (τ_i^*, σ_i^*). We now argue that this outcome is the *unique* Nash equilibrium outcome in Γ.

Suppose that $(\widetilde{\tau}, \widetilde{\sigma})$ is a (possibly mixed) Nash equilibrium where $\widetilde{\tau}_i(s_i)$ is a random variable on $[0, \infty)$ and so is $\widetilde{\sigma}_i(s_i, t_j)$. It is clear that there is no point in randomizing once the other player has exited. Thus, we can write $(\widetilde{\tau}, \sigma)$ where σ is pure.

We first show that if a pure strategy for firm 2 is only weakly dominated in Round 1 (Claim 1 (b)) it cannot be played with positive probability.

Claim 8 *If* $(\widetilde{\tau}, \sigma)$ *is a Nash equilibrium, then* $\Pr[\widetilde{\tau}_2(g_2) < T(b_1, g_2)] = 0$.

Proof. Suppose to the contrary that $\Pr[\widetilde{\tau}_2(g_2) < T(b_1, g_2)] > 0$. We will sub-divide this event into three cases.

Case 1: $\Pr[\widetilde{\tau}_1(g_1) \leq \widetilde{\tau}_2(g_2) < T(b_1, g_2)] > 0$.

In this case, with positive probability g_1 is the first to quit. But for g_1, quitting at any time $t_1 < T(b_1, g_2)$ is strictly worse than quitting at $T(b_1, g_2)$ in expectation. Note that if $s_2 = g_2$, then quitting at t_1 is strictly worse than quitting at $T(b_1, g_2)$. This is because at any time $t < T(b_1, g_2) < T(g_1, b_2)$, the belief of g_1 is such that $p_{1t} > p^*$ (using Lemma 4.1). On the other hand, if $s_2 = b_2$, it is no better.

Case 2: $\Pr[\widetilde{\tau}_2(g_2) < \widetilde{\tau}_1(g_1) < T(b_1, g_2)] > 0$.

In this case, for g_2, quitting at any time $t_2 < T(b_1, g_2)$ is strictly worse than quitting at $T(b_1, g_2)$ in expectation.

Case 3: $\Pr[\widetilde{\tau}_2(g_2) < T(b_1, g_2) \leq \widetilde{\tau}_1(g_1)] > 0$.

Again, for g_2, quitting at any time $t_2 < T(b_1, g_2)$ is strictly worse than quitting at $T(b_1, g_2)$ in expectation.

Thus, we have argued that $(\widetilde{\tau}, \sigma)$ is not a Nash equilibrium. ■

Now we claim that if a pure strategy for firm 1 is only weakly dominated in Round 1 (Claim 1 (a)) it cannot be played with positive probability either.

Claim 9 *If* $(\widetilde{\tau}, \sigma)$ *is a Nash equilibrium, then* $\Pr[\widetilde{\tau}_1(g_1) < T(g_1, b_2)] = 0$.

Proof. Suppose to the contrary that $\Pr[\widetilde{\tau}_1(g_1) < T(g_1, b_2)] > 0$. Again we will sub-divide this event into three cases.

Case 1: $\Pr[\widetilde{\tau}_1(g_1) \leq T(b_1, g_2)] > 0$.

In this case, with positive probability g_1 is the first to quit since by Claim 8, g_2 never quits before $T(b_1, g_2)$. But for g_1 to quit at a time $t_1 < T(g_1, b_2)$ is strictly worse than quitting at $T(g_1, b_2)$ in expectation. This is because if $s_2 = g_2$, this is strictly worse because $\Pr[\widetilde{\tau}_2(g_2) \geq T(b_1, g_2)] = 1$ (Claim 8) and if $s_2 = b_2$, it is no better. Thus, $\Pr[\widetilde{\tau}_1(g_1) \leq T(b_1, g_2)] = 0$.

Case 2: $\Pr[\widetilde{\tau}_2(g_2) < \widetilde{\tau}_1(g_1) \text{ and } T(b_1, g_2) < \widetilde{\tau}_1(g_1) < T(g_1, b_2)] > 0$.

First, note that $\Pr[\widetilde{\tau}_1(b_1) > T(b_1, g_2)] = 0$ as well. This is because from Claim 8, $\Pr[\widetilde{\tau}_2(g_2) \geq T(b_1, g_2)] = 1$ and so when the signals are (b_1, g_2), for b_1 to stay beyond $T(b_1, g_2)$ is strictly worse than dropping out at $T(b_1, g_2)$. When the signals are (b_1, b_2), either dropping out at some $t_1 > T(b_1, g_2)$ is suboptimal because $t_2 \geq t_1$ or it does not matter because $t_2 < t_1$. Thus to drop out at any $t_1 > T(b_1, g_2)$ is suboptimal for b_1.

Now since $\Pr[\widetilde{\tau}_1(b_1) > T(b_1, g_2)] = 0$ and $\Pr[\widetilde{\tau}_1(g_1) \leq T(b_1, g_2)] = 0$ (Case 1), this means that if firm 1 does not quit by time $T(b_1, g_2)$, then firm 2 knows that $s_1 = g_1$. Then it is suboptimal for firm 2 with signal g_2 to drop out at $t_2 < T(g_1, b_2) < T(g_1, g_2)$. When the signals are (b_1, g_2), $t_2 \geq T(b_1, g_2)$ with probability 1 and $t_1 \leq T(b_1, g_2)$ with probability 0. Thus, firm 1 is the first to drop out and thus for g_2 to quit at any $t_2 \geq T(b_1, g_2)$ is irrelevant. Thus, overall firm 2's strategy is not a best response.

Case 3: $\Pr[\widetilde{\tau}_2(g_2) \geq \widetilde{\tau}_1(g_1) \text{ and } T(b_1, g_2) < \widetilde{\tau}_1(g_1) < T(g_1, b_2)] > 0$.

In this case, for g_1 to quit before $T(g_1, b_2)$ is strictly worse than quitting at $T(g_1, b_2)$ in expectation. This is because if $s_2 = g_2$, it is strictly worse and if $s_2 = b_2$ it is no better. ■

So far we have argued that if $(\widetilde{\tau}, \sigma)$ is a (possibly mixed) Nash equilibrium then almost every pure action τ in its support was not weakly dominated in Round 1 of the IEDS procedure. We complete the proof by showing that the same is true in Round 5.

Claim 10 *If* $(\widetilde{\tau}, \sigma)$ *is a Nash equilibrium, then* $\Pr\left[\widetilde{\tau}_2(g_2) < T(g_1, g_2)\right] = 0$.

Proof. Suppose to the contrary that $\Pr\left[\widetilde{\tau}_2(g_2) < T(g_1, g_2)\right] > 0$. Again, we will sub-divide this event into two cases.

Case 1: $\Pr\left[T(g_1, b_2) \leq \widetilde{\tau}_2(g_2) \leq \widetilde{\tau}_1(g_1) < T(g_1, g_2)\right] > 0$.

From Claim 9, $\Pr\left[\widetilde{\tau}_1(g_1) \geq T(g_1, b_2)\right] = 1$ and from Claim 4 $\Pr\left[\widetilde{\tau}_1(b_1) = T(b_1)\right] = 1$. This means that if firm 1 is active at any time $t > T(b_1)$, then with probability 1, firm 2 believes that $s_1 = g_1$. Thus, it is not optimal for g_2 to quit before $T(g_1, g_2)$.

Case 2: $\Pr\left[T(g_1, b_2) \leq \widetilde{\tau}_1(g_1) < \widetilde{\tau}_2(g_2) < T(g_1, g_2)\right] > 0$.

In this case, since Claim 8 implies $\Pr\left[\widetilde{\tau}_2(g_2) \geq T(b_1, g_2)\right] = 1$ and Claim 5 (c) implies $\Pr\left[\widetilde{\tau}_2(b_2) = T(g_1, b_2)\right] = 1$, at any time $t > T(g_1, b_2)$ firm 1 will believe with probability 1 that $s_2 = g_2$. Thus if $\Pr\left[\widetilde{\tau}_1(g_1) > T(g_1, b_2)\right] > 0$, then it is suboptimal for g_1 to quit before $T(g_1, g_2)$. If $\Pr\left[\widetilde{\tau}_1(g_1) = T(g_1, b_2)\right] = 0$, then it is better to stay a little longer and learn whether or not $s_2 = g_2$. ■

The last claim shows that if $(\widetilde{\tau}, \sigma)$ is a Nash equilibrium, the probability that a pure strategy in the support of $\widetilde{\tau}_2(g_2)$ is eliminated in Round 5 of the IEDS procedure is zero.

We have thus argued that no Nash equilibrium can have an outcome different from the one in (τ^*, σ^*).

This completes the proof of Proposition 3. ■

Uniqueness and strict payoff comparison From Proposition 2 we know that the payoffs are strictly ranked if and only if $p(b_1, g_2) > p^*$ or equivalently, $T(b_1, g_2) > 0$. A necessary condition for this is that $\pi > p^*$. Define

$$q_2^- = \min\left\{q_2 \in \left[\frac{1}{2}, q_1\right] : p(b_1, g_2) \geq p^*\right\}$$

Assuming that $\pi > p^*$, for any $q_1 \in \left(\frac{1}{2}, 1\right)$ there exists an open interval $\left(q_2^-, q_2^+\right)$ such that for all $q_2 \in \left(q_2^-, q_2^+\right)$, there is a unique equilibrium in which the payoff ranking is strict (see Figure 4).

In the argument for uniqueness, the only place where we used the fact that q_2 was small relative to q_1 was in Round 3 of the iterated elimination of dominated strategies argument. In that step of the proof, q_2 had to be small enough so that $T(b_1, g_2)$ was close to $T(b_1)$ and the threshold was q_2^+ (which depended on q_1). We will show that $q_2^- < q_2^+$.

If $T(b_1) > 0$, then $T(b_1, g_2) > 0$, the payoff ranking is strict. In this case, $q_2^- = \frac{1}{2}$.

If $T(b_1) = 0$ or equivalently, $p(b_1) \leq p^*$, then since $\pi > p^*$, when $q_2 = q_2^-$, $p(b_1, g_2) = p^*$. In Round 3 of the IEDS argument, when $q_2 = q_2^-$, firm 2 has a strict incentive to wait to see firm 1's signal and so again $q_2^- < q_2^+$.

Multiplicity with near symmetry We have shown that when firm 1's informational advantage is large, there is a unique equilibrium outcome. When this advantage is small, however, there may be other equilibria as well. To see this, suppose that $q_1 - q_2$ is small. Now the argument for uniqueness no longer holds—in particular, the reasoning in Round 3 of the IEDS procedure fails. Indeed, when $q_1 - q_2$ is small enough, there exists an equilibrium which is a "mirror image" of the upstart equilibrium with the roles of firms 1 and 2 interchanged.

In the *mirror equilibrium*, denoted by (τ^{**}, σ^{**}), firm 2's strategy is such that $\tau_2^{**}(b_2) = T(b_2)$ while $\tau_2^{**}(g_2) = T(g_1, g_2)$. Moreover, $\sigma_2^{**}(g_2, T(b_1, g_2)) = T(b_1, g_2)$.

Firm 1's strategy is such that $\tau_1^{**}(b_1) = T(b_1, g_2)$ while $\tau_1^{**}(g_1) = T(g_1, g_2)$. Finally, $\sigma_1^{**}(b_1, T(b_2)) = T(b_2)$ and $\sigma_1^{**}(g_1, T(b_2)) = 2T(g_1, b_2) - T(b_2)$.

Here we have not specified off-equilibrium behavior and beliefs but this can be done by mimicking the upstart equilibrium.

When $q_1 - q_2$ is small, $T(b_1, g_2)$ is close to $T(g_1, b_2)$. Now the arguments confirming that (τ^*, σ^*) is an equilibrium also confirm that (τ^{**}, σ^{**}) is also an equilibrium.

Conditional dominance When q_2 is relatively small, the upstart outcome is not only the unique Nash equilibrium outcome but it is also the unique outcome remaining after iterated elimination of *conditionally dominated* strategies (Shimoji and Watson, 1998).

A strategy for a player is conditionally dominated, if there is an information set for that player that (i) can be reached by the player's own strategy; (ii) is *strictly* dominated by another strategy when measured against only those strategies of other players which can reach the given player's information set. In the iterative procedure carried out above, the strategies that were eliminated in Round 1 and Round 5 were weakly dominated but not strictly dominated. These strategies were, however, conditionally dominated. Thus, the equilibrium outcome we identify is also the only outcome that survives iterated elimination of conditionally dominated strategies.[13]

6 Value of information

In the upstart equilibrium, the startup firm 2 not only wins more often than firm 1, it also obtains a higher equilibrium payoff (Corollary 2). This suggests perhaps that firm 1, say, could be better off with less precise information. This is not the case, however. We show next that despite the fact that the equilibrium payoff of the less-informed firm is higher than that of the better-informed firm, the value of information for both firms is *positive*.[14]

[13] In general games, the iterated elimination of conditionally dominated strategies may leave outcomes that are not Nash equilibria. This is not true in the game considered here, of course.

[14] Bassan et. al (2003) exhibit a simple example where in an otherwise symmetric game, the payoff of the uninformed player 2 is higher than that of the informed player 1. In that game, however, the value of information to player 1 is negative.

Proposition 4 *Suppose $q_1 > q_2$. Then in the upstart equilibrium, firm 1's payoff is increasing in q_1 and firm 2's payoff is increasing in q_2.*

First, consider firm 1. Recall from (10), that

$$\begin{aligned}\Pi_1^* &= \Pr[g_1, g_2] \times v(p(g_1, g_2)) + \Pr[g_1, b_2] \times v(p(g_1, b_2)) \\ &\quad + \Pr[b_1] \times v(p(b_1))\end{aligned}$$

For a fixed q_1, define the probability distribution function $F : [0,1] \to [0,1]$

$$F(x) = \begin{cases} 0 & 0 \le x < p(b_1) \\ \Pr[b_1] & p(b_1) \le x < p(g_1, b_2) \\ \Pr[b_1] + \Pr[g_1, b_2] & p(g_1, b_2) \le x < p(g_1, g_2) \\ 1 & p(g_1, g_2) \le x < 1 \end{cases}$$

and similarly, for $\widehat{q}_1 > q_1$ define $\widehat{F} : [0,1] \to [0,1]$ analogously. It may be readily confirmed that $\widehat{F}$ is a mean-preserving spread of F. Since v is a convex function (see Appendix A), it follows that firm 1's equilibrium payoff when his signal quality is $\widehat{q}_1$, $\widehat{\Pi}_1^* > \Pi_1^*$.

Next, consider firm 2. From (11),

$$\begin{aligned}\Pi_2^* &= \Pr[g_1, g_2] \times v(p(g_1, g_2)) + \Pr[g_1, b_2] \times v(p(g_1, b_2)) \\ &\quad + \Pr[b_1] \times v(p(b_1)) + e^{-rT(b_1)} \Pr[b_1, g_2] \times u\left(p_{T(b_1)}(b_1, g_2)\right)\end{aligned}$$

To establish that Π_2^* is an increasing function of q_2, we will show that the sum of the first two terms is increasing in q_2 and the last term is increasing in q_2 as well. The third term does not depend on q_2.

Lemma 6.1 $\Pr[g_1, g_2] \times v(p(g_1, g_2)) + \Pr[g_1, b_2] \times v(p(g_1, b_2))$ *is increasing in q_2.*

Proof. Since $\Pr[g_1]$ is independent of q_2, it is sufficient to show that

$$\frac{\Pr[g_1, g_2]}{\Pr[g_1]} v(p(g_1, g_2)) + \frac{\Pr[g_1, b_2]}{\Pr[g_1]} v(p(g_1, b_2))$$

is increasing in q_2.

Now if $\widehat{q}_2 > q_2$, then

$$\widehat{p}(g_1, b_2) < p(g_1, b_2) < p(g_1, g_2) < \widehat{p}(g_1, g_2)$$

where $\widehat{p}(g_1, \cdot)$ denotes the posterior derived from $\widehat{q}_2$. Moreover, the mean of $p(g_1, \cdot)$ is $p(g_1)$ and this is the same as the mean of $\widehat{p}(g_1, \cdot)$ (since the expectation of the posteriors is the prior). Thus, the distribution of $\widehat{p}(g_1, \cdot)$ is a mean-preserving spread of the distribution of $p(g_1, \cdot)$.

Since v is a convex function, the result now follows. ∎

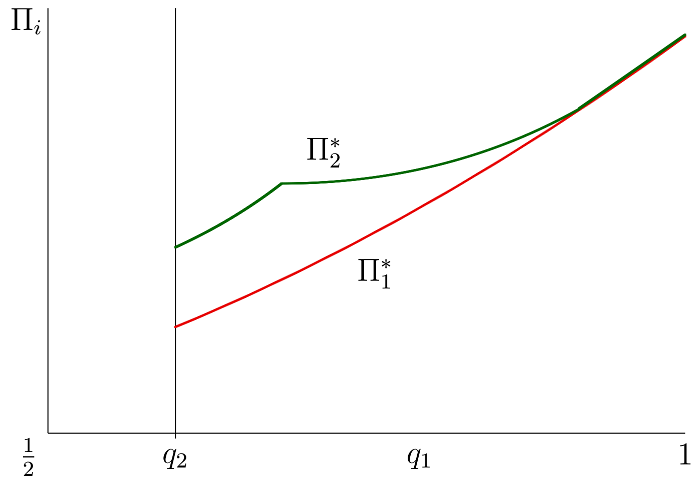

Figure 5: Value of Information

Equilibrium payoffs of both firms are depicted as functions of q_1. The kink in the Π_2^* curve occurs when q_1 is high enough so that $T(b_1) = 0$ and the two curves merge once $T(b_1, g_2) = 0$ as well.

Corollary 2 *Suppose $q_1 > q_2$. Then in the upstart equilibrium, firm 1's payoff is increasing in q_2.*

Lemma 6.2 $\Pr[b_1, g_2] \times u\left(p_{T(b_1)}(b_1, g_2)\right)$ *is increasing in q_2.*

Proof.

$$\begin{aligned}
&\frac{\partial\left(\Pr[b_1, g_2] \times u\left(p_{T(b_1)}(b_1, g_2)\right)\right)}{\partial q_2} \\
&= \frac{\partial \Pr[b_1, g_2]}{\partial q_2} u\left(p_{T(b_1)}(b_1, g_2)\right) + \Pr[b_1, g_2] \frac{\partial p_{T(b_1)}(b_1, g_2)}{\partial q_2} u'\left(p_{T(b_1)}(b_1, g_2)\right) \\
&> \frac{\partial \Pr[b_1, g_2]}{\partial q_2} u\left(p_{T(b_1)}(b_1, g_2)\right) + \Pr[b_1, g_2] \frac{\partial p_{T(b_1)}(b_1, g_2)}{\partial q_2} \frac{u\left(p_{T(b_1)}(b_1, g_2)\right)}{p_{T(b_1)}(b_1, g_2)}
\end{aligned}$$

since u is an increasing and convex function that is non-negative and strictly positive for $p > p^*$ and $u(0) = 0$ (see Appendix A). Thus, $u'(p) > \frac{1}{p} u(p)$. The sign of the right-hand side of the inequality is the same as the sign of

$$\begin{aligned}
&\frac{\partial \Pr[b_1, g_2]}{\partial q_2} p_{T(b_1)}(b_1, g_2) + \Pr[b_1, g_2] \frac{\partial p_{T(b_1)}(b_1, g_2)}{\partial q_2} \\
&= \frac{\partial}{\partial q_2}\left(\Pr[b_1, g_2] \times p_{T(b_1)}(b_1, g_2)\right) \\
&= \frac{\partial}{\partial q_2}\left(\Pr[b_1, g_2] \times \frac{e^{-2\lambda T(b_1)} \pi (1 - q_1) q_2}{e^{-2\lambda T(b_1)} \pi (1 - q_1) q_2 + (1 - \pi) q_1 (1 - q_2)}\right)
\end{aligned}$$

Dividing the numerator and denominator of the second term by $\Pr[b_1, g_2] = \pi(1-q_1)q_2 + (1-\pi)q_1(1-q_2)$, we obtain

$$\begin{aligned} \frac{\partial}{\partial q_2}\left(\Pr[b_1, g_2] \times p_{T(b_1)}(b_1, g_2)\right) &= \frac{\partial}{\partial q_2}\left(\Pr[b_1, g_2] \times \frac{e^{-2\lambda T(b_1)} p(b_1, g_2)}{e^{-2\lambda T(b_1)} p(b_1, g_2) + 1 - p(b_1, g_2)}\right) \\ &= \frac{\partial}{\partial q_2}\left(\frac{e^{-2\lambda T(b_1)} \Pr[G, b_1, g_2]}{1 - \left(1 - e^{-2\lambda T(b_1)}\right) p(b_1, g_2)}\right) \end{aligned}$$

and since both $\Pr[G, b_1, g_2]$ and $p(b_1, g_2)$ are increasing in q_2, we have that $\Pr[b_1, g_2] \times p_{T(b_1)}(b_1, g_2)$ is increasing in q_2 as well. ■

This completes the proof of Proposition 4.

The fact that the value of information is positive for firm 1 does not conflict with the fact that its payoff is lower than that of firm 2. The first is a statement about the derivative of Π_1^* with respect to q_1. The second is a statement comparing the profit levels of the two firms. See Figure 5 which depicts, for fixed q_2, the payoffs Π_1^* and Π_2^* as functions of q_1. Notice that in the upstart equilibrium $\Pi_1^* < \Pi_2^*$ even when $q_1 = q_2$. Of course, as discussed in Section 5, in that case there is a corresponding "mirror" equilibrium as well in which the payoff ranking is reversed.

6.1 Willful ignorance

Proposition 4 shows that firm 1 cannot increase its equilibrium payoff by decreasing the quality of its information while still remaining better informed than firm 2 (and assuming that the upstart equilibrium is played). Precisely, for all $q_2 < q_1' < q_1$,

$$\Pi_1^*(q_1', q_2) < \Pi_1^*(q_1, q_2)$$

where we have now explicitly indicated the dependence of the equilibrium profits on the qualities of the two firms' signals.

But could firm 1 benefit from a drastic decrease in the quality of its information—say, by replacing all its experienced researchers, who have a good idea of the feasibility of the innovation, with new PhDs, who have none—thus becoming the *less*-informed firm? In terms of the model, suppose we start from a situation in which $(q_1, q_2) = (q', q'')$ where $\frac{1}{2} < q'' < q'$ and compare it to a situation in which $(q_1, q_2) = \left(\frac{1}{2}, q''\right)$ so that firm 1 is now less informed than firm 2. In this situation, there is again a unique equilibrium, but this time it is firm 1 which is the upstart.[15] This equilibrium is what we have called a "mirror equilibrium" (see the end of Section 5.2) since the roles of the firms have been reversed. If we denote payoffs in the mirror equilibrium by Π_i^{**}, by symmetry we have (see Figure 6).

$$\Pi_1^{**}\left(\tfrac{1}{2}, q''\right) = \Pi_2^*\left(q'', \tfrac{1}{2}\right)$$

[15] Any attempt to carry out this exercise when there are multiple equilibria is, of course, fraught with peril.

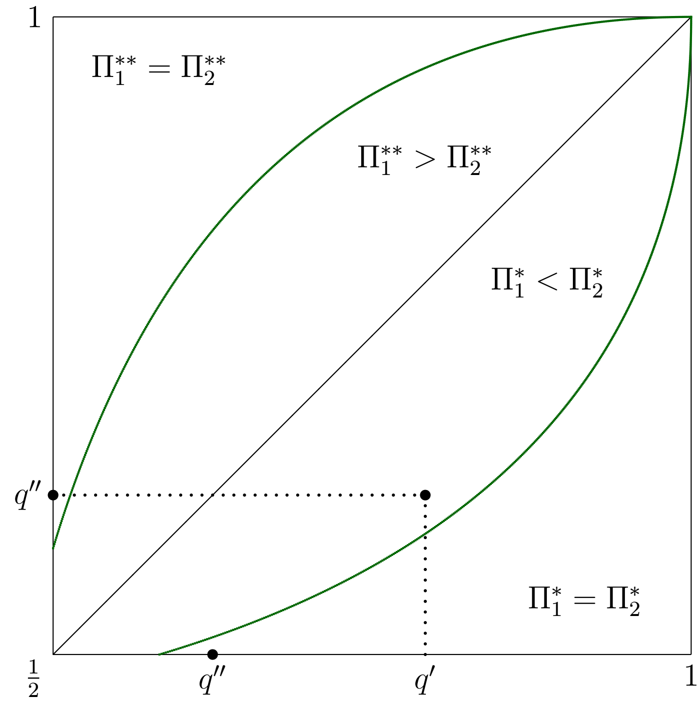

Figure 6: Willful Ignorance

Starting from $(q_1, q_2) = (q', q'')$ firm 1 is worse off by reducing its quality of information to $q_1 = \frac{1}{2}$.

But when the quality of firm 2's information is $\frac{1}{2}$, the upstart equilibrium outcome is unique and the expected profits of the two firms are the same, that is,

$$\Pi_2^*\left(q'', \tfrac{1}{2}\right) = \Pi_1^*\left(q'', \tfrac{1}{2}\right)$$

But in the region where the quality of firm 1's information is higher than that of firm 2, Π_1^* in increasing in *both* qualities (Proposition 4 and Corollary 2). Thus,

$$\Pi_1^{**}\left(\tfrac{1}{2}, q''\right) = \Pi_1^*\left(q'', \tfrac{1}{2}\right) < \Pi_1^*\left(q', q''\right)$$

since $q' > q'' > \frac{1}{2}$. This means that it is not a good idea for the informationally advantaged but competitively disadvantaged firm 1 to become completely uninformed.

Of course, this argument applies not only to the case of complete ignorance, that is, $q_1 = \frac{1}{2}$. As long as, $q_1 > \frac{1}{2}$, is such that $p''(g_1, b_2) \leq p^*$ the same argument applies (here $p''(g_1, b_2) = \Pr[G \mid g_1, b_2]$ computed using qualities q_1 and $q_2 = q''$). This is because the argument above only relies on the equality, $\Pi_2^*(q_1, q'') = \Pi_1^*(q_1, q'')$.

The message of this subsection is: Don't fire the experienced researchers. Willful ignorance does not pay!

7 Many signals

So far we have assumed that each firm's information is binary—there are only two signals. In this section, we show that the main results are robust to the possibility that the firms' information is finer. Suppose that each of the two firms receives one of a finite number of signals, say, $S_1 = \left\{x^1, x^2, ..., x^K\right\}$ and $S_2 = \left\{y^1, y^2, ..., y^L\right\}$. As before, given the state, the signals are conditionally independent. We will assume that the signals can be ordered as $x^k < x^{k+1}$ and $y^l < y^{l+1}$ and that the monotone likelihood property is satisfied, that is,

$$\frac{\Pr\left[x^k \mid G\right]}{\Pr\left[x^k \mid B\right]} \text{ and } \frac{\Pr\left[y^l \mid G\right]}{\Pr\left[y^l \mid B\right]}$$

are strictly increasing in k and l, respectively. As in previous sections, we denote the posterior probabilities as

$$p\left(x^k\right) = \Pr\left[G \mid x^k\right] \text{ and } p\left(y^l\right) = \Pr\left[G \mid y^l\right]$$

and so we have that the posterior probabilities $p\left(x^k\right)$ and $p\left(y^l\right)$ are strictly increasing in k and l, respectively, as well.

We will use the following terminology to describe firm 2's signals.

Definition 2 *A signal y^l is said to be* optimistic *if*

$$\frac{\Pr\left[y^l \mid G\right]}{\Pr\left[y^l \mid B\right]} > 1$$

and pessimistic *if* $\Pr\left[y^l \mid G\right] / \Pr\left[y^l \mid B\right] < 1$.

The monotone likelihood ratio property implies that low signals are pessimistic and high signals optimistic.

In what follows, the following definition will be useful.

Definition 3 *The* quality bound *on the information content of firm 2's signals is*

$$Q_2 = \min\left\{\frac{\Pr\left[y^1 \mid B\right]}{\Pr\left[y^1 \mid G\right]}, \frac{\Pr\left[y^L \mid G\right]}{\Pr\left[y^L \mid B\right]}\right\}$$

Note that since

$$\frac{\Pr\left[y^1 \mid G\right]}{\Pr\left[y^1 \mid B\right]} < 1 < \frac{\Pr\left[y^L \mid G\right]}{\Pr\left[y^L \mid B\right]}$$

it is the case $Q_2 > 1$. To see why this is a measure of information quality, observe that if the quality bound Q_2 is close to 1, then for all l, $p\left(y^l\right) = \Pr\left[G \mid y^l\right]$ is close to π, the prior probability—all the posteriors are close to the prior—and so firm 2's signals are rather uninformative. Also, note that if there were only two signals and $\Pr\left[g_2 \mid G\right] = \Pr\left[b_2 \mid B\right] = q_2$, then $Q_2 = q_2/\left(1 - q_2\right)$.

Now observe that since

$$\frac{p\left(x^k, y^l\right)}{1 - p\left(x^k, y^l\right)} = \frac{p(x^k)}{1 - p\left(x^k\right)} \times \frac{\Pr\left[y^l \mid G\right]}{\Pr\left[y^l \mid B\right]}$$

and we have assumed

$$\frac{p\left(x^{k}\right)}{1-p\left(x^{k}\right)} < \frac{p\left(x^{k+1}\right)}{1-p\left(x^{k+1}\right)}$$

when the quality bound on firm 2's information, Q_2, is close enough to 1, we have

$$p\left(x^{k}, y^{1}\right) < \ldots < p\left(x^{k}, y^{L}\right) < p\left(x^{k+1}, y^{1}\right) < \ldots < p\left(x^{k+1}, y^{L}\right) \tag{13}$$

In other words, firm 2's signals are so poor that they cannot reverse the ranking of posteriors based on firm 1's information alone.

7.1 Upstart equilibrium with many signals

We now demonstrate that, as in Section 4, that there is a perfect Bayesian equilibrium of the R&D race in which firm 2 wins more often than firm 1.

Consider the following strategies. For firm 1,

$$\tau_1^*\left(x^{k}\right) = \begin{cases} T\left(x^{k}\right) & \text{if } k < K \\ T\left(x^{K}, y^{L}\right) & \text{if } k = K \end{cases}$$

$$\sigma_1^*\left(x^{K}, T\left(x^{K}, y^{l}\right)\right) = T\left(x^{K}, y^{l}\right)$$

and

$$\sigma_1^*\left(x^{k}, t_2\right) = 2T\left(x^{k}, y^{L}\right) - t_2 \text{ if } t_2 \neq T\left(x^{K}, y^{l}\right)$$

with the off-equilibrium beliefs that if firm 2 exits at a $t_2 \neq T\left(x^{K}, y^{l}\right)$, then its signal is y^{L}.

For firm 2,

$$\tau_2^*\left(y^{l}\right) = T\left(x^{K}, y^{l}\right)$$

$$\sigma_2^*\left(y^{l}, T\left(x^{k}\right)\right) = \max\left(T\left(x^{k}\right), 2T\left(x^{k}, y^{l}\right) - T\left(x^{k}\right)\right)$$

and

$$\sigma_2^*\left(y^{l}, t_1\right) = 2T\left(x^{K}, y^{l}\right) - t_1$$

with the off-equilibrium beliefs that if firm 1 exits at a $t_1 \neq T\left(x^{k}, y^{L}\right)$, then its signal is x^{K}.

Figure 7 depicts such an equilibrium when $L = 3$. Notice that there are K stages and in stage $k < K$, firm 2 with any signal can learn whether firm 1's signal is x^{k} or higher. Firm 2's information is revealed only in stage K and so only firm 1 with highest signal, x^{K}, can learn firm 2's signal. The learning is severely unbalanced.

We then have

Proposition 1 (M) *There exists a $Q_2^* > 1$ such that if $1 < Q_2 < Q_2^*$, then the strategies (σ^*, τ^*) constitute a perfect Bayesian equilibrium. In this equilibrium, the payoff of the less-informed firm 2 is no less than that of the more-informed firm 1.*

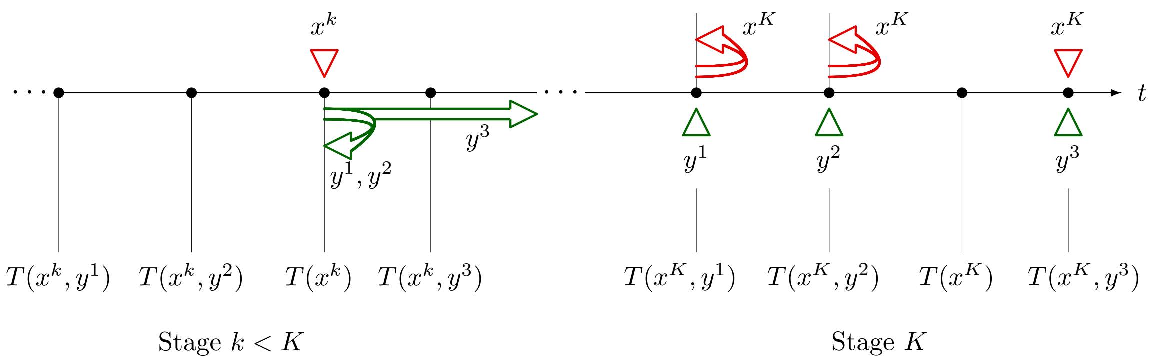

Figure 7: Upstart Equilibrium with Many Signals

Here $L = 3$. There are K stages. In stage $k < K$, firm 1 with signal x^k exits at $T(x^k)$ based only on its own information. Firm 2 follows immediately if it is pessimistic (signal y^1 or y^2) and optimally stays if it is optimistic (signal y^3). The pattern repeats until $k = K - 1$. Firm 2's signal is revealed only in stage K.

Proof. First, suppose that $Q_2 > 1$ is small enough so that (13) holds. This implies that the same ranking holds for $T\left(x^k, y^l\right)$ as well. Precisely,

$$T\left(x^k, y^1\right) \leq ... \leq T\left(x^k, y^L\right) \leq T\left(x^{k+1}, y^1\right) \leq ... \leq T\left(x^{k+1}, y^L\right)$$

and the inequalities are strict unless both sides are 0.

Suppose firm 2 follows (τ_2^*, σ_2^*). Consider firm 1 with signal x^k. Then the argument that it is optimal to unilaterally exit at $\tau_1^*\left(x^k\right)$ is exactly the same as in proof of Proposition 1. Clearly, $\sigma_1^*\left(x^k, \cdot\right)$ is a best response given firm 1's beliefs about y^l.

Now suppose firm 1 follows (τ_1^*, σ_1^*). Consider firm 2 with signal y^l. The proposed strategy τ_2^* asks it to unilaterally drop out at $T\left(x^K, y^l\right)$, that is, it believes that firm 1's signal is x^K, the best possible scenario. Certainly, it cannot be a best response for y^l to choose $\tau_2\left(y^l\right) > T\left(x^K, y^l\right)$. Now consider a $\tau_2\left(y^l\right)$ such that $T\left(x^{K-1}\right) < \tau_2\left(y^l\right) < T\left(x^K, y^l\right)$. In this case, at time $\tau_2\left(y^l\right)$ firm 2 will learn if firm 1's signal is x^K or not. If it is x^K, then firm 1 will not quit before firm 2 quits and it cannot be a best response for 2 to then quit at $\tau_2\left(y^l\right)$. Thus, unilaterally quitting at $\tau_2^*\left(y^l\right) = T\left(x^K, y^l\right)$ is a best response to firm 1's strategy.

If y^l is an optimistic signal (as defined above), then $T\left(x^k, y^l\right) > T\left(x^k\right)$ and so $\sigma_2^*\left(y^l, T\left(x^k\right)\right) = 2T\left(x^k, y^l\right) - T\left(x^k\right)$, which is the optimal quitting time once firm 1 with signal x^k quits at $T\left(x^k\right)$ and firm 2 learns that 1's signal is x^k. But if y^l is a pessimistic signal, then $T\left(x^k, y^l\right) < T\left(x^k\right)$ and so $\sigma_2^*\left(y^l, T\left(x^k\right)\right) > T\left(x^k, y^l\right)$. In this case, once firm 1 with signal x^k quits at $T\left(x^k\right)$ and firm 2 learns that 1's signal is x^k, it quits immediately but suffers from some ex post regret for having stayed too long—in the period between $T\left(x^k, y^l\right)$ and $T\left(x^k\right)$ it loses money. But if Q_2 is close enough to 1, then the gap $T\left(x^k\right) - T\left(x^k, y^l\right)$ is small and the loss is small relative to the gain from learning. Thus, (τ_2^*, σ_2^*) is a best response to (τ_1^*, σ_1^*). ■

Proposition 3 generalizes to the case of many signals as well.

Proposition 3 (M) *There exists a $Q_2^+ \in (1, Q_2^*)$ such that if $1 < Q_2 < Q_2^+$, then the outcome in (σ^*, τ^*) is the unique Nash equilibrium outcome.*

Proof. The proof is very similar to the proof in the case of two signals. The iterated elimination of dominated strategies used to establish Proposition 3 can be mimicked. Here we indicate only the basic steps.

First, as in Round 1, any $\tau_1\left(x^k\right) > T\left(x^k, y^L\right)$ and $\tau_2\left(y^l\right) > T\left(x^K, y^l\right)$ are weakly dominated. In Round 2, any $\tau_1\left(x^k\right) < T\left(x^k, y^1\right)$ is strictly dominated. This means that by staying until $T\left(x^k, y^L\right)$ firm 2 can learn whether firm 1's signal is x^k or whether it is higher. In Round 3, if Q_2 is small enough, $\tau_2\left(y^l\right) < T\left(x^K, y^l\right)$ is strictly dominated. This is because the information of whether 1's signal is x^k or higher comes at the latest by $T\left(x^k, y^L\right)$ and when Q_2 is small, waiting for this information is relatively inexpensive. The remainder of the proof follows that of Proposition 3. ■

8 Unobserved exit

Why is better information a competitive disadvantage in the R&D race studied in this paper? In this section we explore this question by studying a variation of the model of the earlier sections in which a firm's exit decisions are *unobserved* by its rival. In all other respects, the model is the same as outlined in Section 2. If exit is unobserved, then firms can no longer learn each other's private information and now it is no longer the case that better information is a competitive disadvantage. This then serves to isolate the reason why better information is a disadvantage. The better-informed firm has a smaller incentive to learn from the less-informed firm than the other way around and it is this, and only this, that leads to the surprising conclusion that the incumbent firm is at a disadvantage.

With unobserved exit, a strategy for a firm is merely a function $\tau_i : \{b_i, g_i\} \to \mathbb{R}_+$, that is, a pair of unilateral exit times. We will establish

Proposition 5 *With unobserved exit, there is a unique Nash equilibrium $\overline{\tau}$ of the R&D race. In this equilibrium,*

$$\overline{\tau}_1\left(b_1\right) \leq \overline{\tau}_2\left(b_2\right) \leq \overline{\tau}_2\left(g_2\right) \leq \overline{\tau}_1\left(g_1\right)$$

and the payoff of the better-informed firm 1 is greater than that of the less-informed firm 2.

The proof of the proposition is contained in Appendix B and is organized as follows. Lemma B.1 first argues that in any equilibrium with unobserved exit, the exit times must be ranked as in the proposition (all equilibria must be pure). Notice that the ranking of exit times is the same as would occur if firms based their decisions solely on their original beliefs at time 0. The content of the lemma is that even though the firms are updating their prior beliefs as time elapses—a firm is unsure whether its rival is still active—the original ranking of exit times is preserved. Lemma B.2 then shows that there is unique set of equilibrium exit times. When exit is unobserved, uniqueness obtains for *all* parameter values, unlike in the case when exit is observed. The fact that there is a unique equilibrium is somewhat surprising because in effect the game with unobserved exit is a game with *simultaneous* choices and so a backward-induction type of argument cannot be made. Finally, Lemma B.3 shows that at every instant, the expected flow payoff of the better-informed firm is higher than that of the less-informed firm.

9 Planner's problem

How does the upstart equilibrium compare to the solution of a "planner" who seeks to maximize the joint expected profits of the two firms? To analyze such a planner's problem, suppose that the belief that the state is G is $p_0 > p^*$ at time 0.

Since exit is irrevocable and it is never optimal to continue once the belief falls below p^*, the planner's problem reduces to choosing a time S such that both firms are active until $S \leq T \equiv T(p_0)$ and then one of the firms exits. Since both firms engage in R&D until time S, the belief decays at the rate 2λ until S and then at the rate λ after that. Thus per-firm expected flow profit from switching from two firms to one firm at time S is

$$\begin{aligned} w(S) &= \lambda m \int_0^S e^{-rt}\left(e^{-2\lambda t}p_0 + 1 - p_0\right)(p_t - p^*)\,dt \\ &\quad + \frac{1}{2}\lambda m \int_S^{2T-S} e^{-rt}\left(e^{-\lambda(S+t)}p_0 + 1 - p_0\right)(p_t - p^*)\,dt \end{aligned}$$

where the belief p_t at time t that the state is G is defined by

$$\frac{p_t}{1-p_t} = \begin{cases} e^{-2\lambda t}\frac{p_0}{1-p_0} & \text{if } t \leq S \\ e^{-\lambda(S+t)}\frac{p_0}{1-p_0} & \text{if } t \geq S \end{cases} \tag{14}$$

reflecting the fact that both firms are active until time S and after that only one of the two firms is active. Note that $e^{-2\lambda t}p_0 + 1 - p_0$ is the probability that neither firm is successful until time t. Note also that $p_{2T-S} = p^*$ and that the coefficient $\frac{1}{2}$ in the second term appears because w represents per-firm flow profits and the profit of the firm that exits is 0. After substituting for p_t from (14), $w(S)$ can be explicitly calculated to be

$$\begin{aligned} w(S) &= \frac{\lambda m p_0 (1-p^*)}{2\lambda + r}\left((2\lambda + r)e^{-2T\lambda}\left(e^{-rS}-1\right) - r\left(e^{-(2\lambda+r)S}-1\right)\right) \\ &\quad + \frac{\lambda m p_0 (1-p^*)}{2(\lambda + r)}\left(\lambda e^{-2\lambda T}\left(e^{-r(2T-S)} - e^{-rS}\right) - re^{-rS}\left(e^{-2\lambda T} - e^{-2\lambda S}\right)\right) \end{aligned}$$

Differentiating with respect to S then yields

$$w'(S) = \lambda m p_0 (1-p^*) \times \frac{re^{rS}\left(re^{-2(\lambda+r)S} + \lambda e^{-2(\lambda+r)T} - (\lambda + r)e^{-2(\lambda T + rS)}\right)}{2(\lambda + r)}$$

and note that $w'(T) = 0$. Differentiating again we obtain

$$\begin{aligned} & w''(S) \\ = \; & \lambda m p_0 (1-p^*) \times \frac{r^2 e^{rS}\left(\lambda e^{-2(\lambda+r)T} + (\lambda + r)e^{-2(\lambda T + rS)} - (2\lambda + r)e^{-2(\lambda+r)S}\right)}{2(\lambda + r)} \\ < \; & \lambda m p_0 (1-p^*) \times \frac{r^2 e^{rS}\left(\lambda e^{-2(\lambda+r)S} + (\lambda + r)e^{-2(\lambda S + rS)} - (2\lambda + r)e^{-2(\lambda+r)S}\right)}{2(\lambda + r)} \\ = \; & 0 \end{aligned}$$

whenever $S < T$. Thus, w is a concave function and $w'(T) = 0$. As a result, the joint profits of the firms are maximized when $S = T$, that is, when both firms are active until time T. Thus, we obtain that the joint profit-maximizing plan with any initial belief p_0 is for *both* firms to invest in R&D as long as it is profitable, that is, as long as the updated belief $p_t > p^*$ or alternatively, until $T(p_0)$.

How does the upstart equilibrium compare to the planner's optimum?

Recall that in the upstart equilibrium, when the signals are (g_1, b_2), both firms exit at $T(g_1, b_2)$ which is also the planner's optimum exit time. The same is true when the signals are (g_1, g_2). When the signals are (b_1, b_2) the upstart equilibrium may involve too much R&D when $T(b_1, b_2) < T(b_1)$ because in the upstart equilibrium both firms exit at $T(b_1)$ while the planner would want them to exit at $T(b_1, b_2)$. Finally, when the signals are (b_1, g_2) firm 1 exits at $T(b_1)$ while firm 2 invests until time $2T(b_1, g_2) - T(b_1)$. Conditional on (b_1, g_2), the probability of success in this event is then $1 - e^{-2\lambda T(b_1, g_2)}$ and this is the same as that in the planner's solution. Notice that while the overall probability of success in equilibrium is the same as that for the planner, success arrives later in the former case. This is because in equilibrium, when the signals are (b_1, g_2) only one firm exits early—at time $T(b_1)$. This causes "learning-from-failure" to slow down relative to the case when two firms invest, which is the planner's solution.

Proposition 6 *The overall probability of R&D success is higher in the upstart equilibrium than in the planner's optimum.*

Thus, the overall probability of R&D success is higher in the upstart equilibrium than in the planner's optimum—there is too much investment in R&D. If we interpret the planner's problem as arising from a merger of the two firms to form a monopoly and the equilibrium as arising from competition, then this says that competition enhances the chances of R&D success. This runs counter to the sentiments expressed by Schumpeter (1942).

10 Conclusion

We have argued that, somewhat paradoxically, informational asymmetry favors startups over incumbents. This purely informational effect serves to enhance Arrow's replacement effect. The effect appears to be new—it does not stem from a negative value of information or from a second-mover advantage. Rather it stems from the fact that better information diminishes the incentives to learn from one's rival.

At a theoretical level, we have shown that a natural R&D game, in which the only asymmetry is informational, has the following features. There is a unique equilibrium in which information is a competitive disadvantage even though it has positive value. The equilibrium is robust—it is almost the unique rationalizable outcome of the game—and so the finding is not a knife-edge result. Introducing small asymmetries in R&D costs, abilities or the returns to invention would not overturn the results.

A Appendix A: Common beliefs

Although our main concern in this paper is with asymmetric information, in this Appendix we study a situation in which the firms share a common belief about G at time 0. This could, for instance, occur if the signals were publicly known. More important, the firms can learn each other's signal in the course of play in the upstart equilibrium (for instance, if the signals are g_1 and g_2, this occurs at time $T(g_1, b_2)$). At that point, they have common beliefs about the state.

Suppose that the common belief at time 0 is $p_0 > p^*$ and that firm 2 remains active indefinitely. We have seen that the optimal strategy for firm 1 is to remain active until time $T(p_0)$ as defined in (3):

$$e^{-2\lambda T(p_0)} \frac{p_0}{1-p_0} = \frac{p^*}{1-p^*}$$

The expected flow profits of firm 1 are then

$$
\begin{aligned}
v(p_0) &= r\int_0^{T(p_0)} e^{-rt}\left(e^{-2\lambda t}p_0 + 1 - p_0\right)\left(p_t\lambda\frac{m}{r} - c\right)dt \\
&= \lambda m\int_0^{T(p_0)} e^{-rt}\left(e^{-2\lambda t}p_0 + 1 - p_0\right)(p_t - p^*)\,dt \\
&= \lambda m\int_0^{T(p_0)} e^{-rt}\left(e^{-2\lambda t}p_0(1-p^*) - (1-p_0)p^*\right)dt
\end{aligned}
$$

which results in

$$
v(p_0) = \frac{p_0(1-p^*)}{2\lambda + r}\left((2\lambda + r)\,e^{-2T(p_0)\lambda}\left(e^{-rT(p_0)} - 1\right) - r\left(e^{-(2\lambda+r)T(p_0)} - 1\right)\right)
$$

Using the definition of $T(p_0)$ from above, after some calculation we obtain that for $p_0 \geq p^*$,

$$
v(p_0) = -c + \frac{m+2c}{\mu+2}p_0 + \frac{2c}{\mu+2}(1-p_0)\left(\frac{1-p_0}{p_0}\right)^{\frac{\mu}{2}}\left(\frac{p^*}{1-p^*}\right)^{\frac{\mu}{2}} \tag{15}
$$

where $\mu = r/\lambda$. It is easy to see that v is an increasing, convex function. For $p_0 \leq p^*$, $v(p_0) = 0$ since it is optimal for a firm to stay out.

Similarly, if firm 1 were alone and had an initial belief $p_0 > p^*$, the optimal strategy would be to quit at $2T(p_0)$. The single-firm maximized value function in terms of flows is then

$$
u(p_0) = -c + \frac{m+c}{\mu+1}p_0 + \frac{c}{\mu+1}(1-p_0)\left(\frac{1-p_0}{p_0}\right)^{\mu}\left(\frac{p^*}{1-p^*}\right)^{\mu} \tag{16}
$$

Again, u is an increasing, convex function. For $p_0 \leq p^*$, $u(p_0) = 0$. It can be verified that for all $p_0 > p^*$, $u(p_0) > v(p_0)$, that is, competition decreases profits.

When there is no asymmetric information and the beliefs are common, we have

Proposition 7 *With common beliefs, there is a unique Nash equilibrium outcome.*

Proof. With common beliefs, a strategy for firm i is a pair of functions (τ_i, σ_i) as in the main text (now there are no private signals, however). Suppose the common initial belief is p_0. It is easy to see that if firm j chooses $\tau_j = T(p_0)$, then it is a best response for firm $i \neq j$ to choose $\tau_i = T(p_0)$ as well.

To show uniqueness, first note that any (τ_i, σ_i) such that $\tau_i \neq T(p_0)$ is weakly dominated by $(T(p_0), \sigma_i)$. Thus, the Nash equilibrium outcome above is the only outcome that survives one round of elimination of weakly dominated strategies. The argument that there is no Nash equilibrium in weakly dominated strategies is the same as Step 2 in the proof of Proposition 3. ■

B Appendix B: Unobserved exit

In this appendix we collect some results about the model in which firms cannot observe each other's exit decisions.

We begin the analysis by noting that if the rival's exit decisions are unobserved, then there is no possibility of inferring the rival's signal from its behavior. Instead, a firm's beliefs are determined probabilistically by the other firm's strategy. No matter what behavior is postulated for its rival, firm i's belief $p_t(s_i)$ *strictly decreases* over time.[16]

The fact that, given the rival's behavior, $p_t(s_i)$ is a decreasing function of t means that when the rival's exit cannot be observed, a firm should exit at the earliest time that its belief $p_t(s_i) \leq p^*$. This in turn implies that *all equilibria of the game with unobserved exit must be pure.* In order to mix between two exit times t and $t' > t$, say, it must be that $p_t(s_i) = p_{t'}(s_i)$ which is impossible.

To see how the beliefs are determined, suppose firm j follows the strategy τ_j and suppose that $\tau_j(b_j) < \tau_j(g_j)$. At any time $t \in [\tau_j(b_j), \tau_j(g_j)]$, firm i knows that b_j would have exited while g_j would still be active. If firm $i \neq j$ is active at t, then its posterior belief $p_t(s_i)$ at time t must satisfy

$$\begin{aligned}\frac{p_t(s_i)}{1-p_t(s_i)} &= \frac{\Pr[G \mid s_i, \mathcal{S}_0(t)]}{\Pr[B \mid s_i, \mathcal{S}_0(t)]} \\ &= \frac{\Pr[s_i \mid G]\Pr[G]}{\Pr[s_i \mid B]\Pr[B]} \times \frac{\Pr[\mathcal{S}_0(t) \mid G]}{\Pr[\mathcal{S}_0(t) \mid B]} \\ &= \frac{p(s_i)}{1-p(s_i)} \times \frac{\Pr[\mathcal{S}_0(t) \mid G]}{1} \\ &= \frac{p(s_i)}{1-p(s_i)} \times e^{-\lambda t}\left(q_j e^{-\lambda t} + (1-q_j)\, e^{-\lambda \tau_j(b_j)}\right) \qquad (17)\end{aligned}$$

In the last expression of (17), the first $e^{-\lambda t}$ is the probability that firm i has not succeeded until t. The expression in parentheses is the probability that firm j has not succeeded until t either. This is a weighted average of the probability that g_j has not succeeded until t and the probability that b_j did not succeed before exiting at $\tau_j(b_j)$.

At any time $t < \tau_j(b_j)$, the corresponding expression in the parentheses would be $e^{-\lambda t}$ also since both b_j and g_j would be active. At any time $t > \tau_j(g_j)$, the expression would be $q_j e^{-\lambda \tau_j(g_j)} + (1-q_j)\, e^{-\lambda \tau_j(b_j)}$ since both b_j and g_j would have exited by then.

We first establish that in any equilibrium the exit times must be ordered in a particular way.

[16] This is in constrast to the model of Section 3 in which a rival's exit decision is observed. There the absence of exit indicates that the other firm's information is favorable and so may lead to an increase in the posterior probabilility of state G.

Lemma B.1 *Any equilibrium* (τ_1, τ_2) *of the R&D race with unobserved exit must satisfy*

$$\tau_1(b_1) \leq \tau_2(b_2) \leq \tau_2(g_2) \leq \tau_1(g_1)$$

and the inequalities are strict unless both sides are zero.

Proof. First, given τ_j it is the case that for all t, $p_t(b_i) < p_t(g_i)$ and so $\tau_i(b_i) \leq \tau_i(g_i)$. Moreover, if $\tau_i(b_i) > 0$, then $\tau_i(b_i) < \tau_i(g_i)$.

Claim 1: $\tau_1(b_1) \leq \tau_2(b_2)$. To see this, suppose to the contrary that $\tau_2(b_2) < \tau_1(b_1)$. Then, at time $t = \tau_2(b_2)$

$$\frac{p(b_2)}{1 - p(b_2)} e^{-2\lambda\tau_2(b_2)} \leq \frac{p^*}{1 - p^*}$$

But because $p(b_1) < p(b_2)$, we have

$$\frac{p(b_1)}{1 - p(b_1)} e^{-2\lambda\tau_2(b_2)} < \frac{p^*}{1 - p^*}$$

as well and so $\tau_1(b_1) > \tau_2(b_2)$ is not possible since if it is unprofitable for b_2 to continue, it is also unprofitable for b_1 to continue. Thus, $\tau_1(b_1) \leq \tau_2(b_2)$.

Claim 2: $\tau_2(b_2) < \tau_1(g_1)$. Again, suppose not. Then, given that $\tau_1(b_1) \leq \tau_1(g_1)$, we have

$$\tau_1(b_1) \leq \tau_1(g_1) \leq \tau_2(b_2) < \tau_2(g_2)$$

and since both b_1 and g_1 exit before both b_2 and g_2, we have for $s_1 = b_1, g_1$

$$\frac{p(s_1)}{1 - p(s_1)} e^{-2\lambda\tau_1(s_1)} \leq \frac{p^*}{1 - p^*} \tag{18}$$

The belief of b_2 at time $t = \tau_1(g_1)$ must be (using (17))

$$\frac{p_t(b_2)}{1 - p_t(b_2)} = \frac{p(b_2)}{1 - p(b_2)} e^{-\lambda\tau_1(g_1)} \left(q_1 e^{-\lambda\tau_1(g_1)} + (1 - q_1) e^{-\lambda\tau_1(b_1)}\right)$$

and using (18) we have that

$$e^{-2\lambda\tau_1(g_1)} \leq \frac{p^*}{1 - p^*} \frac{1 - q_1}{q_1} \frac{1 - \pi}{\pi} \text{ and } e^{-2\lambda\tau_1(b_1)} \leq \frac{p^*}{1 - p^*} \frac{q_1}{1 - q_1} \frac{1 - \pi}{\pi}$$

and so

$$e^{-\lambda\tau_1(b_1)} e^{-\lambda\tau_1(g_1)} \leq \frac{p^*}{1 - p^*} \frac{1 - \pi}{\pi}$$

Now at $t = \tau_1(g_1)$ we have

$$\begin{aligned}
\frac{p_t(b_2)}{1 - p_t(b_2)} &\leq \frac{1 - q_2}{q_2} \frac{\pi}{1 - \pi} \left(q_1 \frac{p^*}{1 - p^*} \frac{1 - q_1}{q_1} \frac{1 - \pi}{\pi} + (1 - q_1) \frac{p^*}{1 - p^*} \frac{1 - \pi}{\pi}\right) \\
&= \frac{1 - q_2}{q_2} 2(1 - q_1) \frac{p^*}{1 - p^*} \\
&< \frac{p^*}{1 - p^*}
\end{aligned}$$

since $\frac{1}{2} < q_2 < q_1$. But this means that b_2 would not want to stay after $\tau_1(g_1)$, which contradicts the assumption that $\tau_1(g_1) \leq \tau_2(b_2)$.

Claim 3: $\tau_2(g_2) \leq \tau_1(g_1)$. Again, suppose not. Then from the arguments above we have

$$\tau_1(b_1) \leq \tau_2(b_2) \leq \tau_1(g_1) < \tau_2(g_2)$$

We will show that at time $t = \tau_1(g_1)$,

$$p_t(g_1) > p_t(g_2)$$

Using (17), the required inequality is equivalent to

$$\frac{q_1}{1-q_1}\left((1-q_2)\, e^{-\lambda\tau_2(b_2)} e^{-\lambda t} + q_2 e^{-2\lambda t}\right) > \frac{q_2}{1-q_2}\left((1-q_1)\, e^{-\lambda\tau_1(b_1)} e^{-\lambda t} + q_1 e^{-2\lambda t}\right)$$

and this is the same as

$$\frac{(1-q_2)^2}{q_2} e^{-\lambda\tau_2(b_2)} - \frac{(1-q_1)^2}{q_1} e^{-\lambda\tau_1(b_1)} > (q_2 - q_1)\, e^{-\lambda t} \tag{19}$$

Since the right-hand side is negative, it is enough to show that the left-hand side is positive.

If $0 = \tau_1(b_1) = \tau_2(b_2)$, then we are done, so suppose $0 \leq \tau_1(b_1) < \tau_2(b_2)$. Then these exit times satisfy

$$\begin{aligned}
\frac{p(b_1)}{1-p(b_1)} e^{-2\lambda\tau_1(b_1)} &\leq \frac{p^*}{1-p^*} \\
\frac{p(b_2)}{1-p(b_2)} e^{-\lambda\tau_2(b_2)} \left(q_1 e^{-\lambda\tau_2(b_2)} + (1-q_1)\, e^{-\lambda\tau_1(b_1)}\right) &= \frac{p^*}{1-p^*}
\end{aligned}$$

The second condition defines $e^{-\lambda\tau_2(b_2)}$ as the positive solution to a quadratic equation. Now observe that if we have a quadratic equation of the form $Ax^2 + Bx = C$, where A, B and C are positive, then the positive solution is decreasing in B. Thus, the exit times must satisfy

$$e^{-\lambda\tau_1(b_1)} \leq \sqrt{\beta\frac{q_1}{1-q_1}} \tag{20}$$

$$e^{-\lambda\tau_2(b_2)} \geq \frac{1}{2q_1}\left(-\sqrt{\beta q_1(1-q_1)} + \sqrt{\beta q_1(1-q_1) + 4q_1\beta\frac{q_2}{1-q_2}}\right) \tag{21}$$

where to economize on notation we write $\beta = \frac{p^*}{1-p^*}\frac{1-\pi}{\pi}$.

The left-hand side of (19) is positive if

$$\frac{(1-q_2)^2}{q_2} e^{-\lambda\tau_2(b_2)} > \frac{(1-q_1)^2}{q_1} e^{-\lambda\tau_1(b_1)}$$

and using the bounds from (20) and (21), it is sufficient to show that

$$\frac{(1-q_2)^2}{q_2}\frac{1}{2q_1}\left(-\sqrt{\beta q_1(1-q_1)}+\sqrt{\beta q_1(1-q_1)+4q_1\beta\frac{q_2}{1-q_2}}\right) > \frac{(1-q_1)^2}{q_1}\sqrt{\beta\frac{q_1}{1-q_1}}$$

and after some routine algebra we obtain that the required inequality is equivalent to

$$1 > q_2\left(\frac{1-q_1}{1-q_2}\right)^3 + (1-q_1)\left(\frac{1-q_1}{1-q_2}\right)$$

which holds. ■

Uniqueness

Lemma B.2 *There is a unique equilibrium $(\overline{\tau}_1, \overline{\tau}_2)$ of the R&D race with unobserved exit.*

Proof. Let $(\overline{\tau}_1, \overline{\tau}_2)$ be unique solution to the following recursive system:

$$\begin{aligned}
\overline{\tau}_1(b_1) &= T(b_1) \\
\overline{\tau}_2(b_2) &= \min\left\{t \geq 0 : \frac{p(b_2)}{1-p(b_2)}e^{-\lambda t}\left((1-q_1)e^{-\lambda\overline{\tau}_1(b_1)} + q_1 e^{-\lambda t}\right) \leq \frac{p^*}{1-p^*}\right\} \\
\overline{\tau}_2(g_2) &= \min\left\{t \geq 0 : \frac{p(g_2)}{1-p(g_2)}e^{-\lambda t}\left((1-q_1)e^{-\lambda\overline{\tau}_1(b_1)} + q_1 e^{-\lambda t}\right) \leq \frac{p^*}{1-p^*}\right\} \\
\overline{\tau}_1(g_1) &= \min\left\{t \geq 0 : \frac{p(g_1)}{1-p(g_1)}e^{-\lambda t}\left((1-q_2)e^{-\lambda\overline{\tau}_2(b_2)} + q_2 e^{-\lambda\overline{\tau}_2(g_2)}\right) \leq \frac{p^*}{1-p^*}\right\}
\end{aligned}$$

Notice that left-hand side of each of the defining inequalities is a strictly decreasing function of t and so there is at most one solution. We will argue that the unique solution $(\overline{\tau}_1, \overline{\tau}_2)$ satisfies the condition in the Lemma B.1. From the definitions above, it is then immediate that these exit times are optimal (see 17).

Claim 1: $\overline{\tau}_1(b_1) \leq \overline{\tau}_2(b_2)$ and the inequality is strict unless both are zero.

If $\overline{\tau}_1(b_1) = 0$, there is nothing to prove and so suppose $\overline{\tau}_1(b_1) > 0$. At $t = \overline{\tau}_1(b_1)$,

$$\begin{aligned}
\frac{p(b_2)}{1-p(b_2)}e^{-\lambda\overline{\tau}_1(b_1)}\left((1-q_1)e^{-\lambda\overline{\tau}_1(b_1)} + q_1 e^{-\lambda\overline{\tau}_1(b_1)}\right) &= \frac{p(b_2)}{1-p(b_2)}e^{-2\lambda\overline{\tau}_1(b_1)} \\
&> \frac{p(b_1)}{1-p(b_1)}e^{-2\lambda\overline{\tau}_1(b_1)}
\end{aligned}$$

and so $\overline{\tau}_2(b_2) > \overline{\tau}_1(b_1)$.

Claim 2: $\overline{\tau}_2(b_2) \leq \overline{\tau}_2(g_2)$ and the inequality is strict unless both are zero.

Again, if $\overline{\tau}_2(b_2) = 0$, there is nothing to prove so suppose that $\overline{\tau}_2(b_2) > 0$. Now from the fact that $p(b_2) < p(g_2)$, it follows immediately that $\overline{\tau}_2(b_2) < \overline{\tau}_2(g_2)$.

Claim 3: $\overline{\tau}_2(g_2) \leq \overline{\tau}_1(g_1)$ and the inequality is strict unless both are zero.

As before, it is enough to consider the case where $\overline{\tau}_2(g_2) > 0$. We will argue that at time $t = \overline{\tau}_2(g_2)$, firm g_1 wants to continue or that

$$\begin{aligned} & \frac{p(g_1)}{1-p(g_1)} e^{-\lambda\overline{\tau}_2(g_2)} \left((1-q_2) e^{-\lambda\overline{\tau}_2(b_2)} + q_2 e^{-\lambda\overline{\tau}_2(g_2)} \right) \\ > \; & \frac{p(g_2)}{1-p(g_2)} e^{-\lambda\overline{\tau}_2(g_2)} \left((1-q_1) e^{-\lambda\overline{\tau}_1(b_1)} + q_1 e^{-\lambda\overline{\tau}_2(g_2)} \right) \end{aligned} \tag{22}$$

and this is the same as

$$\frac{(1-q_2)^2}{q_2} e^{-\lambda\overline{\tau}_2(b_2)} - \frac{(1-q_1)^2}{q_1} e^{-\lambda\overline{\tau}_1(b_1)} > (q_2 - q_1) e^{-\lambda\overline{\tau}_2(g_2)}$$

and the it is enough to show that the left-hand side is positive. The proof that this is true is exactly the same as in the proof of Lemma B.1. ■

Equilibrium payoffs

Lemma B.3 *In the unique equilibrium of the R&D with unobserved exit, the expected payoff of firm* 1 *is greater that that of firm* 2.

Proof. Firm i's expected flow payoff at time t is

$$\overline{\Pi}_i^t = \Pr[b_i] \, \overline{\Pi}_i^t(b_i) + \Pr[g_i] \, \overline{\Pi}_i^t(b_i)$$

where $\overline{\Pi}_i^t(s_i)$ denotes the flow payoff at time t when firm i's signal is s_i. We will argue that for all t, $\overline{\Pi}_1^t \geq \overline{\Pi}_2^t$ and for some interval of time, the inequality is strict.

Now when active, the flow payoff is (see (6))

$$\begin{aligned} \overline{\Pi}_i^t(s_i) &= r \Pr[\mathcal{S}_0(t)] \left(p_t(s_i) \frac{\lambda m}{r} - c \right) \\ &= \lambda m \Pr[\mathcal{S}_0(t)] \left(p_t(s_i) - p^* \right) \end{aligned}$$

where, as usual, $\mathcal{S}_0(t)$ denotes the event that there has been no success until time t. Thus, firm s_i's flow payoff at time t when active is

$$\overline{\Pi}_i^t(s_i) = \Pr[\mathcal{S}_0(t) \mid G] \Pr[s_i \mid G] \Pr[G] (1-p^*) - \Pr[s_i \mid B] \Pr[B] \, p^*$$

because

$$\begin{aligned} \Pr[s_i] \Pr[\mathcal{S}_0(t) \mid s_i] &= \Pr[\mathcal{S}_0(t), s_i \mid G] \Pr[G] + \Pr[\mathcal{S}_0(t), s_i \mid B] \Pr[B] \\ &= \Pr[\mathcal{S}_0(t) \mid G] \Pr[s_i \mid G] \Pr[G] + \Pr[s_i \mid B] \Pr[B] \end{aligned}$$

and

$$\Pr[s_i] \Pr[\mathcal{S}_0(t) \mid s_i] \, p_t(s_i) = \Pr[\mathcal{S}_0(t) \mid G] \Pr[s_i \mid G] \Pr[G]$$

Some routine calculations then show that firm 1's expected flow payoff is then

$$\overline{\Pi}_1^t = \lambda m \times \pi (1-p^*) \times \begin{cases} e^{-2\lambda t} - \beta & t \in [0, \overline{\tau}_1(b_1)] \\ q_1 e^{-2\lambda t} - (1-q_1)\beta & t \in [\overline{\tau}_1(b_1), \overline{\tau}_2(b_2)] \\ q_1 e^{-\lambda t}\left(q_2 e^{-\lambda t} + (1-q_2) e^{-\lambda \overline{\tau}_2(b_2)}\right) - (1-q_1)\beta & t \in [\overline{\tau}_2(b_2), \overline{\tau}_2(g_2)] \\ q_1 e^{-\lambda t}\left(q_2 e^{-\lambda \overline{\tau}_2(g_2)} + (1-q_2) e^{-\lambda \overline{\tau}_2(b_2)}\right) - (1-q_1)\beta & t \in [\overline{\tau}_2(g_2), \overline{\tau}_2(g_1)] \end{cases}$$

where, as above, $\beta = \frac{p^*}{1-p^*}\frac{1-\pi}{\pi}$. Similarly, firm 2's expected flow payoff is

$$\overline{\Pi}_2^t = \lambda m \times \pi (1-p^*) \times \begin{cases} e^{-2\lambda t} - \beta & t \in [0, \overline{\tau}_1(b_1)] \\ e^{-\lambda t}\left(q_1 e^{-\lambda t} + (1-q_1) e^{-\lambda \overline{\tau}_1(b_1)}\right) - \beta & t \in [\overline{\tau}_1(b_1), \overline{\tau}_2(b_2)] \\ q_2 e^{-\lambda t}\left(q_1 e^{-\lambda t} + (1-q_1) e^{-\lambda \overline{\tau}_1(b_1)}\right) - (1-q_2)\beta & t \in [\overline{\tau}_2(b_2), \overline{\tau}_2(g_2)] \\ 0 & t \in [\overline{\tau}_2(g_2), \overline{\tau}_2(g_1)] \end{cases}$$

We will show that for all t, $\overline{\Pi}_1^t \geq \overline{\Pi}_2^t$ and for a positive interval of time, the inequality is strict.

Case 1: $t \in [0, \overline{\tau}_1(b_1)]$.

In this interval, $\overline{\Pi}_1^t = \overline{\Pi}_2^t$.

Case 2: $t \in [\overline{\tau}_1(b_1), \overline{\tau}_2(b_2)]$.

In this region, the inequality $\overline{\Pi}_1^t \geq \overline{\Pi}_2^t$ is equivalent to

$$q_1 \beta \geq e^{-\lambda t}(1-q_1) e^{-\lambda \overline{\tau}_1(b_1)}$$

Because the right-hand side is decreasing in t, it is enough to show that

$$q_1 \beta \geq (1-q_1) e^{-2\lambda \overline{\tau}_1(b_1)}$$

and since $\beta = \frac{p^*}{1-p^*}\frac{1-\pi}{\pi}$, this is equivalent to

$$\frac{p^*}{1-p^*} \geq \frac{p(b_1)}{1-p(b_1)} e^{-2\lambda \overline{\tau}_1(b_1)}$$

which defines $e^{-2\lambda \overline{\tau}_1(b_1)}$.

Case 3: $t \in [\overline{\tau}_2(b_2), \overline{\tau}_2(g_2)]$

In this region, the inequality $\overline{\Pi}_1^t \geq \overline{\Pi}_2^t$ is easily seen to be equivalent to

$$(q_1 - q_2)\beta > q_1 q_2 e^{-\lambda t}\left(\frac{1-q_1}{q_1} e^{-\lambda \overline{\tau}_1(b_1)} - \frac{1-q_2}{q_2} e^{-\lambda \overline{\tau}_2(b_2)}\right) \tag{23}$$

The left-hand side of the inequality is positive and we claim that the right-hand side is negative. To see this, first note that, $\overline{\tau}_2(b_2) < 2T(b_2) - T(b_1)$ because at

$t = 2T(b_2) - T(b_1)$ firm 2's belief is

$$\begin{aligned}
& \frac{p(b_2)}{1-p(b_2)} e^{-\lambda(2T(b_2)-T(b_1))} \left((1-q_1) e^{-\lambda T(b_1)} + q_1 e^{-\lambda(2T(b_2)-T(b_1))}\right) \\
= & \frac{p(b_2)}{1-p(b_2)} e^{-2\lambda T(b_2)} \left((1-q_1) + q_1 e^{-2\lambda T(b_2)} e^{2\lambda T(b_1)}\right) \\
\leq & \frac{p^*}{1-p^*} \left((1-q_1) + q_1 e^{-2\lambda(T(b_2)-T(b_1))}\right) \\
\leq & \frac{p^*}{1-p^*}
\end{aligned}$$

since $T(b_2) \geq T(b_1)$. This means that firm b_2 does not want to stay after $2T(b_2) - T(b_1)$.

Since $\overline{\tau}_1(b_1) = T(b_1)$ and $\overline{\tau}_2(b_2) < 2T(b_2) - T(b_1)$, the right-hand side of (23) is less than

$$\begin{aligned}
& q_1 q_2 e^{-\lambda t} \left(\frac{1-q_1}{q_1} e^{-\lambda T(b_1)} - \frac{1-q_2}{q_2} e^{-\lambda(2T(b_2)-T(b_1))}\right) \\
= & q_1 q_2 e^{-\lambda t} e^{\lambda T(b_1)} \left(\frac{1-q_1}{q_1} e^{-2\lambda T(b_1)} - \frac{1-q_2}{q_2} e^{-2\lambda T(b_2)}\right)
\end{aligned}$$

which is negative by the definitions of $T(b_1)$ and $T(b_2)$. ■

References

[1] Arrow, K. (1962): "Economic Welfare and the Allocation of Resources for Invention," in *The Rate and Direction of Inventive Activity*, R. Nelson (ed.), Princeton University Press, Princeton, NJ.

[2] Bassan, B., O. Gossner, M. Scarsini, and S. Zamir (2003): "Positive Value of Information in Games," *International Journal of Game Theory*, 32 (1), 17–31.

[3] Benoît, J. P., & Dubra, J. (2011): "Apparent Overconfidence," *Econometrica*, 79 (5), 1591–1625.

[4] Chatterjee, K. and R. Evans (2004): "Rivals' Search for Buried Treasure: Competition and Duplication in R&D," *Rand Journal of Economics*, 35 (1), 160–183.

[5] Chen, C-H. and J. Ishida (2017): "A War of Attrition with Experimenting Players," Osaka University, https://ideas.repec.org/p/dpr/wpaper/1014.html

[6] Choi, J. (1991): "Dynamic R&D Competition under 'Hazard Rate' Uncertainty," *Rand Journal of Economics,* 22 (4), 596–610.

[7] Christiansen, C. (1997): *The Innovator's Dilemma: When New Technologies Cause Great Firms to Fail*, Harvard Business School Press, Boston, MA.

[8] Das, K. and N. Klein (2018): "Competition, Search and Duplication in a Patent Race," Department of Economics, University of Exeter, http://people.exeter.ac.uk/kd320/duplicative_search_1.pdf

[9] Dong, M. (2018): "Strategic Experimentation with Asymmetric Information," Penn State University, https://sites.google.com/site/miaomiaodong0/1-delimit

[10] Gilbert, R. and D. Newberry (1982): "Preemptive Patenting and the Persistence of Monopoly," *American Economic Review*, 76, 238–242.

[11] Hirshleifer, J. (1971): "The Private and Social Value of Information and the Reward to Inventive Activity," *American Economic Review*, 61, 561–574.

[12] Igami, M. (2017); "Estimating the Innovator's Dilemma: Structural Analysis of Creative Destruction in the Hard Disk Industry, 1981–1998," *Journal of Political Economy*, 125, 798–847.

[13] Keller, G., S. Rady and M. Cripps (2005): "Strategic Experimentation with Exponential Bandits," *Econometrica*, 73, 39–68.

[14] Klein, N. and P. Wagner (2018): "Strategic Investment and Learning with Private Information," University of York, https://sites.google.com/site/peterachimwagner/research

[15] Maschler, M., E. Solan and S. Zamir (2013): *Game Theory*, New York: Cambridge University Press.

[16] Malueg, D. and S. Tsutsui (1997): "Dynamic R&D Competition with Learning," *Rand Journal of Economics*, 28, 751–772.

[17] Moscarini, G. and F. Squintani (2010): "Competitive Experimentation with Private Information: The Survivor's Curse," *Journal of Economic Theory*, 145, 639–660.

[18] Rietveld, N., P. Groenen, P. Koellinger, M. Loos, and R. Thurik (2013): "Living Forever: Entrepreneurial Overconfidence at Older Ages," Erasmus Research Institute of Management, http://hdl.handle.net/1765/40673.

[19] Schumpeter, J. (1942): *Capitalism, Socialism and Democracy*, New York: Harper.

[20] Shimoji, M. and J. Watson (1998): "Conditional Dominance, Rationalizability and Game Forms," *Journal of Economic Theory*, 83, 161–195.

[21] Surowiecki, J. (2014): "Epic Fails of the Startup World," *The New Yorker*, May 19.

[22] Wong, T-N. (2018); "Free Riding and Duplication in R&D," University of Basel, https://sites.google.com/site/tszningwongecon/research

[23] Wu, B., and A. Knott (2006): "Entrepreneurial Risk and Market Entry," *Management Science,* 52 (9), 1315–1330.